

# Phil 202

## SAMPLE First Exam

You may use your book and notes in answering these questions.

I. Symbolize the following statements, using the sentential letters indicated:

1. If Terry and Douglas both outrun House (T,D), Bramlett will be left open (B) unless Edgerson fouls out (E), but then Lute's hair will turn white (L). (10 points)
2. If the bank robber escaped in either a Chevy (C) or a Ford (F), but Howard was caught with the money in a Toyota (T), then either Carole hid the money there (H) or Mary found it in the garbage (M) and gave it to Howard to return (R). (10 points)

II. Construct a truth table for the following formula, and report whether it is a tautology (10 points):

$$[(P \ \& \ \sim Q) \leftrightarrow \sim(P \rightarrow Q)]$$

III. Prove the following metatheorem (10 points):

For any formulas  $A$  and  $B$ , if  $\vdash(A \vee B)$  then  $\sim A \vdash B$ .

(Note: you must give a general argument. You cannot do this by constructing a truth table.)

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### Solutions

You may use your book and notes in answering these questions.

I. Symbolize the following statements, using the sentential letters indicated:

1. If Terry and Douglas both outrun House (T,D), Bramlett will be left open (B) unless Edgerson fouls out (E), but then Lute's hair will turn white (L). (10 points)

If T and D, then B unless E, but (if E) then L  
 If (T & D) then  $[(\sim E \rightarrow B) \& (E \rightarrow L)]$   
 $\{(T \& D) \rightarrow [(\sim E \rightarrow B) \& (E \rightarrow L)]\}$

2. If the bank robber escaped in either a Chevy (C) or a Ford (F), but Howard was caught with the money in a Toyota (T), then either Carole hid the money there (H) or Mary found it in the garbage (M) and gave it to Howard to return (R). (10 points)

If C or F, but T, then either H or (M and R).  
 If  $[(C \vee F) \& T]$  then  $[H \vee (M \& R)]$   
 $\{[(C \vee F) \& T] \rightarrow [H \vee (M \& R)]\}$

II. Construct a truth table for the following formula, and report whether it is a tautology (10 points):

P	Q	$[(P \& \sim Q) \leftrightarrow \sim(P \rightarrow Q)]$				
T	T	F	F	T	F	T
T	F	T	T	T	T	F
F	T	F	F	T	F	T
F	F	F	T	T	F	T

It is a tautology

III. Prove the following metatheorem (10 points):

For any formulas  $A$  and  $B$ , if  $\vdash (A \vee B)$  then  $\sim A \vdash B$ .

(Note: you must give a general argument. You cannot do this by constructing a truth table.)

If  $\vdash (A \vee B)$  then for every interpretation, either  $A$  is true or  $B$  is true. Thus for any assignment relative to which  $\sim A$  is true,  $A$  is false and hence  $B$  is true. That means that  $\sim A \vdash B$ .

Many students reasoned as follows: If  $(A \vee B)$  is true then either  $A$  is true or  $B$  is true. Thus if  $A$  is false,  $B$  is true. What this shows is that relative to any interpretation, if  $(A \vee B)$  is true and  $\sim A$  is true then  $B$  is true. But what this means is that  $(A \vee B), \sim A \vdash B$ , which is not the desired metatheorem.

