Part Four: Epistemic Cognition, Focussing First on Deductive Reasoning

- Epistemic reasoning is driven by both input from perception and queries passed from practical cognition.
- The way in which epistemic interests effect the course of cognition is by initiating backward reasoning.
- Example of bidirectional reasoning

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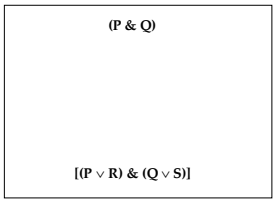

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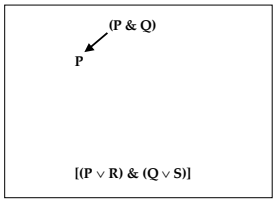

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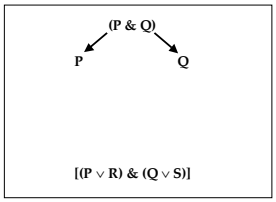

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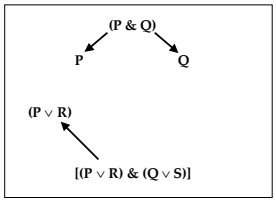

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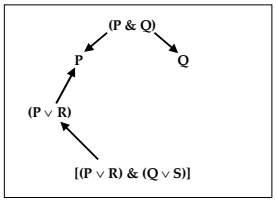

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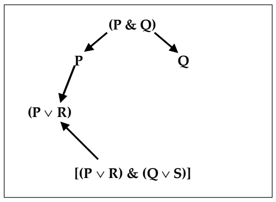

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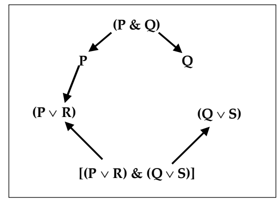

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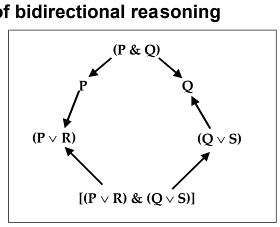

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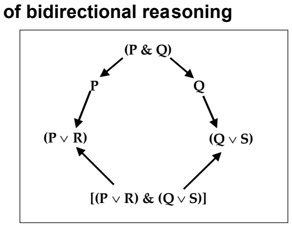

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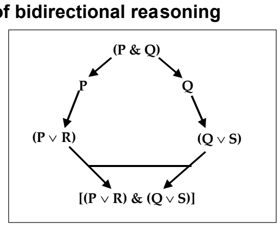

OSCAR as a Deductive Reasoner

- OSCAR’s greatest virtue as an automated reasoner is that it is capable of performing defeasible reasoning. However, the defeasible reasoner is built on top of a deductive reasoner, and is best understood by looking at the deductive reasoner first.

Natural Deduction

- **OSCAR's reasoning is in the style of "natural deduction".**
  - **I take this to mean that it reasons about what follows from the premises *given suppositions*.**
  - **This is implemented by having OSCAR reason about *sequents*. These are pairs <*supposition*,*formula*> where *supposition* is a set of formulas. Abbreviated *formula* / *supposition*.**
  - **The most characteristic rule of suppositional reasoning is *CONDITIONALIZATION*:**
    Given an interest in (P → Q)/X suppose {P} and adopt interest in inferring Q/{P}∪X.
  - **Another example, *DILEMMA*:**
    Given (P v Q) and an interest in R/X, adopt interest in R/X∪{P} and R/X∪{Q}.

Bidirectional Reasoning

- **Perhaps the most novel feature of OSCAR's deductive reasoning is that reason-schemas are segregated into backward and forward schemas.**
  - **forward schemas lead from conclusions to conclusions**
    From (P & Q), infer P.
  - **backward schemas lead from interests to interests**
    From P, Q infer (P & Q).

Some Inference Rules

adjunction

$$\frac{p/X \quad q/Y}{(p\&q)/X\cup Y}$$

simplification

$$\frac{(p\&q)/X}{p/X \quad q/X}$$

negation introduction

$$\frac{p/X}{\sim\sim p/X}$$

negation elimination

$$\frac{\sim\sim p/X}{p/X}$$

addition

$$\frac{p/X}{(pvq)/X \quad (qvp)/X}$$

disjunctive syllogism

$$\frac{\sim p/X,\ (pvq)/Y}{q/Y} \qquad \frac{\sim q/X,\ (pvq)/Y}{p/X}$$

conditionalization

$$\frac{q/X\cup\{p\}}{(p \rightarrow q)/X}$$

modus ponens

$$\frac{p/X,\ (p \rightarrow q)/Y}{q/X\cup Y}$$

modus tollens

$$\frac{\sim q/X,\ (p \rightarrow q)/Y}{\sim p/X\cup Y}$$

reductio1

$$\frac{p/X\cup\{\sim p\}}{p/X}$$

reductio2

$$\frac{(q\ \&\ \sim q)/X\cup\{\sim p\}}{p/X}$$

Directionality

- **Most of these inference rules have natural directions, and are combinatorially explosive when applied in the opposite direction.**
- **A plausible classification:**

Forwards reasons	Backwards reasons
» simplification	adjunction
» negation elimination	negation introduction
» disjunctive syllogism	addition
» modus ponens	conditionalization
» modus tollens	reductio1

- **Note that reductio2 fits neither category. I will return to this.**

Interest-Driven Reasoning

- **We can think of interest-driven reasoning as consisting of three operations:**
  - **(1) we reason forwards from previously drawn conclusions to new conclusions;**
  - **(2) we reason backwards from interests to interests;**
  - **(3) when we have reasoned backwards to a set of sequents as interests and forwards to the same set of sequents as conclusions, then we discharge interest and conclude the sequent that led to those interests.**

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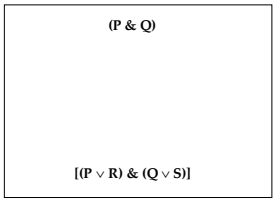

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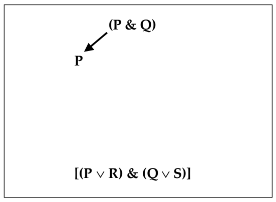

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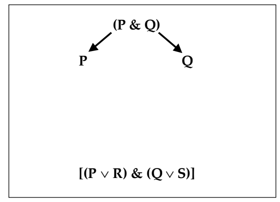

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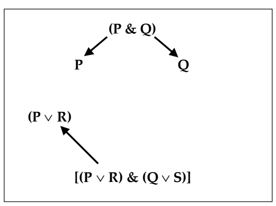

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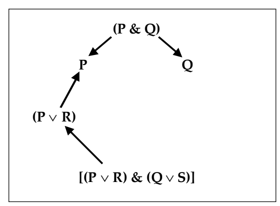

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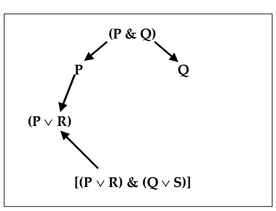

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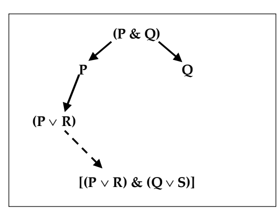

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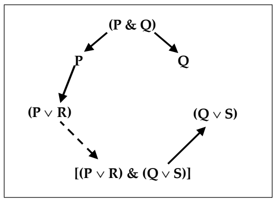

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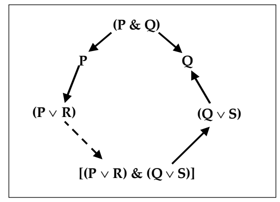

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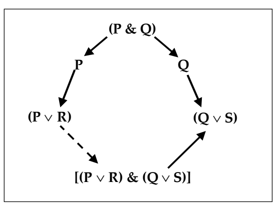

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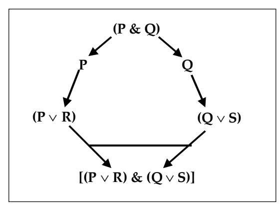


Interest-Driven Reasoning

- reason-forwards
  - If a set of sequents X is a forwards reason for a sequent S, some member of X is newly concluded, and the other members of X have already been concluded, then conclude S.
- reason-backwards
  - If interest is adopted in a sequent S, and a set X of sequents is a backwards reason for S, then adopt interest in any members of X that have not already been concluded. If every member of X has been concluded, conclude S.
- discharge-interest
  - If interest was adopted in the members of X as a way of getting the sequent S, and some member of X is concluded and the other members of X have already been concluded, then conclude S.

Generalized Backwards Reasons

- This is inadequate for reductio2:

reductio2

$$\frac{(q \,\&\, {\sim}q)/X \cup \{{\sim}p\}}{p/X}$$

The proper interpretation of this rule should be:

Given an interest in p/X, suppose {~p}. Then for each conclusion q/X∪{~p} drawn relative to the reductio supposition, adopt interest in ~q/X∪{~p}. When such a contradiction is concluded, conclude p/X.

Generalized backwards reasons have both forwards and backwards premises:

example:

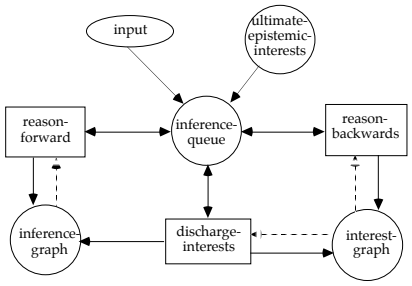
$$(\forall x)(Fx \rightarrow Gx)/X$$
$$\frac{Fa/X}{Ga/X}$$

Generalized Forward Reasons

- **We can allow forwards reasons to have backwards premises as well as forwards premises. The intent is that once the forwards premises have been instantiated by conclusions, we adopt interest in the backwards premises.**
- **reason-forwards**
  - **If a triple ⟨X,Y,S⟩ instantiates a forward reason-schema, some member of X is newly concluded, and the other members of X have already been concluded, then adopt interest in the first member of Y that has not already been concluded. If every member of Y has been concluded, conclude S.**

- **reason-backwards**
  - **Given a new interest in a sequent S such that for some X,Y, the triple ⟨X,Y,S⟩ instantiates a backward reason-schema and all members of X have already been concluded, then adopt interest in the first member of Y that has not already been concluded. If every member of Y has been concluded, conclude S. If some members of X have not been concluded, then simply record X,Y as a potential reason for S, for use by discharge-interest.**
- **discharge-interest**
  - **If ⟨X,Y,S⟩ instantiates a backward reason-schema, interest has been adopted in S, some member of X is newly concluded and all other members of X have already been concluded, adopt interest in the first member of Y that has not already been concluded. If every member of Y has been concluded, conclude S.**
  - **If ⟨X,Y,S⟩ instantiates a forward reason-schema, all members of X have already been concluded, and some member of Y is newly concluded, adopt interest in the first member of Y that has not already been concluded. If every member of Y has been concluded, conclude S.**

Interest-Driven Reasoning



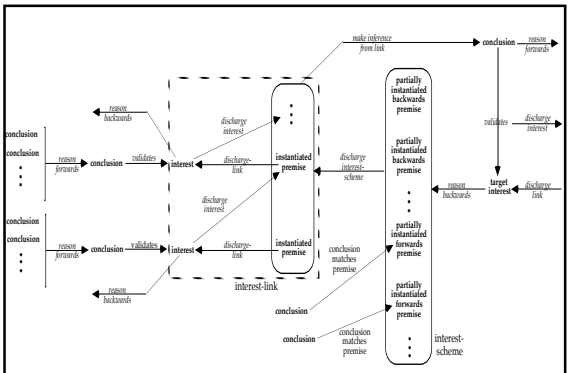


Figure 14. Backwards reasoning and the discharge of interest-links.

Defining Reason-Schemas

(def-forwards-reason symbol
:forwards-premises *list of formulas*
:backwards-premises *list of formulas*
:conclusions *list of formulas*
:variables *list of symbols*)

(def-backwards-reason symbol
:conclusions *list of formulas*
:forwards-premises *list of formulas*
:backwards-premises *list of formulas*
:variables *list of symbols*)

Defining Reason-Schemas

(def-forwards-reason MODUS-PONENS
:conclusions Q
:forwards-premises
P
(P → Q)
:variables P Q)

(def-backwards-reason ADDITION
:conclusions (P&Q)
:backwards-premises
P
Q
:variables P Q)

examples of reasoning in the propositional calculus

Quantifiers — Instantiation Rules

Forwards reasons:

quantifier negation eliminations:

infer $(\exists x)\neg P$ from $\sim(\forall x)P$

infer $(\forall x)\neg P$ from $\sim(\exists x)P$

universal instantiation:

infer $Sb(c,x)P/X$ from $(\forall x)P/X$ where c is a term already occurring in some conclusion Q/Y such that $Y \subseteq X$ and $Sb(c,x)P$ results from substituting c for all free occurrences of x in P. If there are no such terms, infer $Sb(@,x)P/X$ from $(\forall x)P/X$.

existential instantiation:

infer $Sb(@x,x)P/X$ from $(\exists x)P/X$ where $@x$ is a constant that has not previously occurred in any conclusions.

Auxiliary rule for forwards reasoning:

If Q/Y is a newly adopted conclusion, then for each conclusion of the form $(\forall x)P/X$ such that $Y \subseteq X$, infer $Sb(c,x)P/X$ from $(\forall x)P/X$ where c is a term occurring in Q/Y but not occurring in any previous conclusions.

Quantifiers - instantiation rules

Backwards reasons:

quantifier negation introductions:

adopt interest in $(\exists x)\neg P$ to infer $\sim(\forall x)P$

adopt interest in $(\forall x)\neg P$ to infer $\sim(\exists x)P$

universal generalization:

adopt interest in $Sb(\hat{}x,x)P/X$ to infer $(\forall x)P/X$, where $\hat{}x$ is a free variable that has not previously occurred in any conclusions.

existential generalization:

adopt interest in $Sb(c,x)P/X$ to infer $(\exists x)P/X$ where c is a term already occurring in some conclusion Q/Y such that $Y \subseteq X$. If there are no such terms, adopt interest in $Sb(@,x)P/X$ to infer $(\exists x)P/X$.

Auxiliary rule for backwards reasoning:

If Q/Y is a newly adopted conclusion, then for each interest of the form $(\exists x)P/X$ such that $Y \subseteq X$, adopt interest in $Sb(c,x)P/X$ to infer $(\exists x)P/X$ where c is a term occurring in Q/Y but not occurring in any previous conclusions.

Quantifiers — Skolemization and Unification

- In forwards-reasoning, universally bound variables are instantiated by free variables (this is the rule UI), and existentially bound variables are instantiated by skolem-functions whose arguments are all the free variables already occurring in the formula (this is EI).
- In backwards-reasoning, existentially bound variables are instantiated by free variables (this is the rule EG), and universally bound variables are instantiated by skolem-functions whose arguments are all the free variables already occurring in the formula (this is UG).
- Forwards reasoning and interest-discharge then use unification.

Deductive Reasoning in OSCAR

- OSCAR is surprisingly efficient as a deductive reasoner.
  - In a recent comparison with the the highly respected OTTER resolution-refutation theorem prover on a set of 163 problems chosen by Geoff Sutcliffe from the TPTP theorem proving library:
    - » OTTER failed to get 16
    - » OSCAR failed to get 3
    - » On problems solved by both theorem provers, OSCAR (written in LISP) was on the average 40 times faster than OTTER (written in C)
- OSCAR's advantage lies in its startling efficiency in proof-search.

examples

- Completeness and Soundness of Natural Deduction