The Formal Study of Reasoning

1. Problem Solving and Reasoning

Human beings are unique in their ability to acquire sophisticated knowledge about the way the world works and then use that knowledge to achieve their goals. In using their knowledge and figuring out how to achieve a goal, they are engaging in problem solving. Nonhuman animals also engage in problem solving, but they are quite limited in the complexity of the problems they can solve. Although my cat may be able to solve the problem of catching a bird by climbing a tree, she will never figure out how to send a rocket to the moon.

Consider a simple instance of problem solving. This is something chimpanzees can do. The chimp is put in a cage with a large box, and bananas are hanging from the ceiling. The chimp wants the bananas. The problem is how to get them. The chimp solves the problem by moving the box under the bananas and climbing on top of it, from which it can reach the bananas. We can only conjecture how chimps solve this problem, but humans solve the same problem by reasoning. Using our knowledge of the world, we observe that the bananas are in a certain location. We know that we can push the box under the bananas, we know that we can climb on top of the box, and it was beneath the bananas then we could reach the bananas. Finally, we know that if we could reach the bananas, we could retrieve them. So we reason that if we push the box beneath the bananas and climb on top of it, then we can retrieve the bananas. Our reasoning is as follows:

1. If I push the box beneath the bananas, then it will be beneath the bananas.
2. If the box is beneath the bananas and I climb on top of it, then I will be able to reach the bananas.
3. If I am able to reach the bananas, then I can retrieve them.

Therefore, if I push the box beneath the bananas and climb on top of it, then I can retrieve the bananas.

The reasoning we employed in solving this problem can be generalized. It has a general structure that can be revealed by abbreviating “I push the box beneath the bananas” as P, “the box will be beneath the bananas” as B, “I climb on top of the box” as C, “I am able to reach the bananas” as A, and “I can retrieve the bananas” as R:
1. If P, then B.
2. If B and C, then A.
3. If A, then R.

Therefore, if P and C, then R.

What should be noticed is that this reasoning would be equally good reasoning if we let P, B, C, A, and R abbreviate anything else. It is the general structure of the reasoning that makes it good reasoning.

To solve more complex problems, we may have to engage in more complex reasoning. For instance, if I want to see a movie, and the only way I can get there is by taxi, I may reason as follows:

1. If I am at the movie theatre and I buy a ticket, I can see the movie.
2. If I am at the movie theatre and I have seven dollars, I can buy a ticket.
3. If I am at home and I pay a cab driver ten dollars and ask him to drive me to the theatre, he will do so.
4. If the cab driver drives me to the theatre, then I will be at the theatre.
5. If I pay the cab driver ten dollars, then I will have ten dollars less than I did to begin with.

Therefore, if I am at home and have seventeen dollars, I pay a cab driver ten dollars and ask him to drive me to the theatre, and I buy a ticket, I can see the movie.

This reasoning is much like the reasoning involved in retrieving the bananas, except that it also involves some mathematical reasoning.

Solving different kinds of problems may require different kinds of knowledge about the world and different kinds of reasoning. If my goal is to send a rocket to the moon, I will have to know a lot about physics and engineering, and my reasoning will be heavily mathematical. If my goal is to be elected mayor of Tucson, I will need different kinds of knowledge and the reasoning will also be different. If my goal is to write a logic text, the required knowledge and reasoning will be different yet.

2. Rationality, Logic, and Reasoning

One of the traditional topics of philosophy is rational thought. We want to know how it works. How is it possible for us to acquire sophisticated knowledge of the world and use that to solve complex problems? Reasoning is an important part of rational thought, so the study of good reasoning is part of the study of rationality.

2.1 Varieties of Reasoning
We engage in many different kinds of reasoning:

- Some of our reasoning is “perceptual”. If I look at a book on my desk in full view and in good light and it looks red to me, then I can reasonably conclude that (probably) it is red. Most of our knowledge of the world begins with such perceptual reasoning.
- Another kind of reasoning is inductive. In inductive reasoning, we draw general conclusions from a number of observations of particular instances. For example, most people would concur that earthworms don’t have wings. Why do they believe that? Presumably, because they have seen many earthworms, and none of those they have seen have had wings. So they reason to the general conclusion that earthworms don’t have wings from their beliefs about individual earthworms.

- We cannot directly observe the mental states of other people, but we form beliefs about their mental states by observing their behavior and comparing it with our own when we are in various mental states.
- We form scientific theories by constructing explanations for observations. For instance, the atomic theory of matter was originally propounded to explain diverse physical phenomena such as the peculiar spectra of light that substances emit when heated.

- We often engage in mathematical reasoning. For example, if Jones gives me five dollars and Smith gives me seven, I can infer that together they gave me twelve dollars.

These varieties of reasoning are characterized in part by their subject matter. Perceptual reasoning has to do with making inferences from perceptual input. Inductive reasoning has to do with reasoning about generalizations. Reasoning about the mental states of others is about other minds. And so on. The field of philosophy that studies these diverse kinds of reasoning is epistemology. Epistemologists try to clarify the way each of these kinds of reasoning works, and explain why they are examples of good reasoning.

2.2 Formally Good Reasoning

Logic is also about reasoning, but focuses on a different aspect of it. Sometimes, the goodness of a bit of reasoning has nothing to do with its subject matter. The analogous reasoning is correct when applied to any subject matter. We saw an example of this above. No matter what P, B, C, A, and R might be, the following reasoning is good reasoning:

1. If P, then B.
2. If B and C, then A.
3. If A, then R.

Therefore, if P and C, then R.

It is the form of the reasoning that makes it good rather than its content. Logic is the study of this phenomenon. Consider another example. The following is clearly good reasoning:
1. All Republicans are faithful to their spouses.
2. No one who is faithful to his or her spouse will ever get into political trouble.

Therefore, no Republican will ever get into political trouble.

One may doubt the truth of the premises, but the reasoning is impeccable. That is, given the premises, one can infer the conclusion. The correctness of this inference, however, has nothing to do with the fact that it is about Republicans, spouses, and political trouble. The inference has the general form:

1. All A are B.
2. No B is C.

Therefore, no A is C.

Here is another instance of the same form of reasoning:

1. All Republicans are unfaithful to their spouses.
2. No one who is unfaithful to his or her spouse will ever be elected President.

Therefore, no Republican will ever be elected President.

This reasoning is equally good, and for the same reason. Again, one might object to the premises, but given the premises, the conclusion follows.

Reasoning that is good because of the form of the reasoning can be called formally good reasoning. Logic is the study of formally good reasoning. In logic we investigate what forms of reasoning generate formally good reasoning.

2.3 Soundness and Inferential Correctness

The preceding examples illustrate an important distinction. We can evaluate reasoning in two different ways. Reasoning begins from premises and draws a conclusion. One way to evaluate the reasoning is by asking whether the premises it assumes are true. Neither of the above arguments fares well on this dimension of evaluation. But we can also ask whether, given the premises, the argument supports the conclusion. On this dimension of evaluation, both of the preceding arguments are flawless.

Let us say that reasoning is inferentially correct when it is good on the second dimension. Inferentially correct reasoning is reasoning wherein the inferences are correct, regardless of whether the premises are true. All of the reasoning we have discussed so far has been inferentially correct. However, some of that reasoning has proceeded from false premises. Reasoning is said to be sound when it is inferentially correct reasoning from true premises. Sound reasoning is reasoning that survives evaluation on both of the above dimensions.

Logic does not concern itself with soundness—only with inferential
correctness. To know whether reasoning is sound, we must investigate the truth of the premises, and depending upon what the premises are, that may take us far afield from philosophy into physics, history, economics, geography, or sometimes just common sense. Inferential correctness, on the other hand, can be investigated without engaging in scientific investigations of the world. In exploring the concept of inferential correctness, we can ignore the truth or falsity of the premises and just ask whether the conclusion would follow if the premises were true.

The terminology “sound argument” is traditional in logic, but it is also unfortunate. As just noted, we will not have occasion to talk about sound arguments, so the terminology is not useful. To make matters worse, when we get further into the study of logic, we will find that the term “sound” has other uses unrelated to its use in talking about sound arguments, and those other uses are more important.

3. Arguments and Inferences

Reasoning proceeds by making inferences. We start with some premises, and end with a conclusion, but typically there will be a number of steps in between. Consider the following puzzle:

A murder has been committed, and an eyewitness reports that

1. The murderer was either John or Joe,

but they are identical twins so the eyewitness could not tell them apart. The detectives have discovered the following clues:

2. The murder was committed in Tucson.
3. One of the brothers was in Phoenix at the time of the crime, but the murderer was in Tucson.
4. John was in Phoenix only if Joe had breakfast with Maria on the morning of the crime.
5. Joe had a fight with Maria the night before, and would not have had breakfast with Maria unless they made up.
6. Maria would only make up with Joe if John asked her to.
7. John only asked Maria to make up with Joe if John was in Tucson.

We also know:

8. If someone was in Tucson at a certain time, they were not in Phoenix at that same time.

Who was the murderer?

We can solve this puzzle by reasoning from the premises in the following way:
9. It follows from (1) that if John wasn’t the murderer, Joe was.
10. It follows from (3) that either John was in Phoenix or Joe was in Phoenix.
11. Suppose John is not the murderer. Then we can reason as follows:
12. From (11) it follows that Joe is the murderer.
13. From (2) and (12) it follows that Joe was in Tucson.
14. From (8) and (13) it follows that Joe was not in Phoenix.
15. From (3) and (14) it follows that John was in Phoenix.
16. From (4) and (15) it follows that Joe had breakfast with Maria.
17. From (5) and (16) it follows that Joe made up with Maria.
18. From (6) and (17) it follows that John asked Maria to make up with Joe.
19. From (7) and (18) it follows that John was in Tucson.
20. From (3) and (19) it follows that Joe was in Phoenix, and hence was not the murderer.
21. From (1) and (20) it follows that John is the murderer.
22. So from the supposition that John is not the murderer, it follows that he is. Hence that supposition cannot be true, so:

23. John is the murderer.

Consider how this puzzle was solved. We began with premises (1)–(8), and we ended with the conclusion (23). However, it is not initially obvious that the conclusion follows from the premises, so we fill the gap with a number of intermediate inferences each of which is obvious by itself. Stringing those inferences together allows us to establish the conclusion. This illustrates that reasoning is essentially a tool whose purpose is to enable us to acquire new knowledge on the basis of things we already know.

When we represent our reasoning in this way, what we write is an argument. An argument is a list of statements each of which is either a premise or inferred from earlier steps of the argument. The individual steps represent either premises or inferences from previous steps. It is convenient to number the steps of the argument, and include an explanation of how each statement is inferred.

**Exercises.** Construct arguments solving the following puzzles:

**First Puzzle:**
When Arthur, Bill, and Carter eat out:
1. Each orders either ham or pork (but not both).
2. If Arthur orders ham, Bill orders pork.
3. Either Arthur or Carter orders ham, but not both.
4. Bill and Carter do not both order pork.
Who could have ordered ham one day and pork another day?
Second Puzzle:
Fred knows five women: Ann, Betty, Carol, Deb, and Eve.
1. The women are in two age brackets: three women are under 30, and two women are over 30.
2. Two women are teachers and the other three women are secretaries.
3. Ann and Carol are in the same age bracket.
4. Deb and Eve are in different age brackets.
5. Betty and Eve have the same occupation.
6. Carol and Deb have different occupations.
7. Of the five women, Fred will marry the teacher over thirty.
8. Each of these women is either a secretary or a teacher but not both.
9. Fred will marry one of these women.
Whom will Fred marry?

4. Two Kinds of Reasoning

4.1 Deductive Reasoning
There is a distinction between two kinds of reasoning. Some reasoning has the characteristic that if the premises are true then the conclusion could not fail to be true. For such reasoning, there are no conceivable circumstances under which the premises are true but the conclusion false. Most of the arguments we have considered have this characteristic. For example, recall the argument:

1. All Republicans are faithful to their spouses.
2. No one who is faithful to his or her spouse will ever get into political trouble.

Therefore, no Republican will ever get into political trouble.

We can certainly imagine circumstances in which the conclusion is false. In fact, the real world is probably one such circumstance. But we cannot imagine circumstances in which the conclusion is false while the premises are true. The only way the conclusion of this argument could be false is by having one or more premises false as well. Such an argument is deductive. In a deductive argument, the conclusion follows of necessity from the premises.

4.2 Defeasible Reasoning
Not all good reasoning proceeds in terms of deductive arguments. Section two briefly surveyed a number of different kinds of arguments that are important to our being able to acquire knowledge of the world, and most of those arguments are non-deductive. For example, consider inductive reasoning. One might reason as follows:
I have observed many earthworms, and none have had wings.

Therefore, no earthworms have wings.

It is clear that this is not a deductive argument. The fact that you have not observed any earthworms with wings does not make it impossible for there to be some. Europeans once reasoned similarly about swans, concluding that all swans are white. But it turned out that there are black swans in Australia.

It is a general characteristic of inductive reasoning that it is not deductive. In fact, that is the whole point of it. The value of inductive reasoning is that it allows us to discover general truths about the world. We can never observe general truths. All we can do is observe particular instances of them, and infer what the general truths are on the basis of the particular instances. This is what science is all about.

There is a temptation to suppose that if inductive reasoning is not deductive, then there is something wrong with it. But that is to mistake what the purpose of reasoning is. Reasoning is a tool of rationality, and what it does is lead us to reasonable beliefs. The beliefs must be reasonable, but they need not be infallible. None of our interesting beliefs about the world are infallible. Try to think of anything you know that you know on the basis of premises that deductively guarantee its truth. The only examples you are apt to find are mathematical examples or simple tautologies like “It is snowing or it isn’t snowing”. Virtually all of the copious knowledge you have of the world and in terms of which you successfully solve practical problems is had on the basis of reasoning that is not deductive.

Good arguments that are not deductive make their conclusions reasonable, but do not absolutely guarantee their truth. When we employ such arguments, we must always be prepared to retract the conclusions if subsequent discoveries mandate that. Such arguments are said to be defeasible, because they can be “defeated”. The considerations that defeat a defeasible inference are called defeaters. For example, consider the earthworm argument. What might a defeater be? Here is an obvious example:

This is an earthworm and it has wings.

In other words, no matter how many confirming instances we have for a generalization, a single counter-example suffices to show it false and defeat the inductive inference. This is how the reasoning supporting the generalization “All swans are white” was defeated.

Most of our everyday reasoning is defeasible. For example, when I infer that the book on my desk is red because it looks red to me, I am reasoning defeasibly. That is, I could be wrong without being wrong about whether it looks red to me. When I infer that my wife is angry because she is yelling at me and brandishing an umbrella, I could be wrong—she might be pretending. When physicists proposed the Bohr atom as the best expla-
nination for various phenomena in atomic physics, they were wrong, as we now know. That is, their inference was a reasonable one, but subsequent discoveries have forced the retraction of their conclusion.

We do employ deductive reasoning, but it is usually interspersed with defeasible reasoning. For example, consider the argument to the effect that John is the murderer. That argument is deductive, but where do the premises come from? The first premise was to the effect that the murderer was either John or Joe, and was based upon eyewitness testimony. Clearly, such testimony provides only a defeasible basis for an inference. The same is true of the other premises of that argument. The picture of human reasoning that emerges from this is one in which we engage in a lot of defeasible reasoning, drawing conclusions on the basis of perceptual input and induction, and then once we have a fair amount of information to go on, we do some deductive reasoning.

This suggests that if we want to study reasoning, we will learn more by studying defeasible reasoning rather than deductive reasoning. That is probably true, but defeasible reasoning is harder to study than deductive reasoning and less well understood. It is also easier to understand the way defeasible reasoning works against a background understanding of deductive reasoning. So the bulk of this book will be concerned with deductive reasoning, and only at the end will we return to an examination of defeasible reasoning.

5. Deductive Inference

5.1 Logical Possibility

It is customary to try to define deductive inference in terms of logical possibility. This is the way we proceeded when we introduced the term in section four. We said that a deductive inference is one in which the conclusion could not possibly be false while the premises are true.

There are many kinds of possibility. There is an “epistemic” sense in it is possible that the President is now in Honduras but it is not possible that he is now on Mars. This is epistemic in the sense that, “for all we know”, he might be on Honduras, but our knowledge rules out the possibility of his being on Mars.

There is also a non-epistemic sense in which, even when we know that the President is currently in Washington, we are willing to say that the President could now be in Honduras but could not now be on Mars. This has to do with “practical possibility”. The President could have gone to Honduras, but he could not have gone to Mars even if he wanted to.

Another variety of possibility is physical possibility. Something is physically possible just in case it is not ruled out by fundamental laws of nature. For example, the special theory of relativity tells us that nothing can move faster than the velocity of light, so traveling faster than light is physically impossible. On the other hand, traveling at 99.999% the velocity
of light is physically possible, but certainly not practically possible.

Physical possibility is a very weak requirement. Almost anything we can think of is physically possible, even if it isn't practically possible. However, there is an even weaker kind of possibility. Traveling faster than light is physically impossible, but the laws of nature might have been other than they are, in which case something could travel faster than light. The sense in which the laws of nature might have been other than they are is logical possibility. Logical possibility is the weakest kind of possibility. All that it precludes are those things that could not, in any sense, be true.

A popular way of trying to explain logical possibility is in terms of possible worlds. A possible world is “a way the world could be”. A possible world is specified by giving a complete description of a world, where that description is logically possible. Then something is logically possible just in case there is a possible world in which it is true. Of course, this explanation is circular, because we need the notion of logical possibility to define the notion of a possible world. Nevertheless, this is sometimes a helpful way of thinking about logical possibility.

5.2 Logical Necessity and Validity

Logical necessity is defined in terms of logical possibility. Something is logically necessary just in case it is not logically possible for it to be false. Equivalently, something is logically necessary just in case it is true in all possible worlds. Logical necessities are things that “couldn’t be false”, in the strongest possible sense of “couldn’t”.

It is customary to characterize deductive inference in terms of logical necessity or logical possibility. An inference is deductive just in case it is logically impossible for the premises to be true while the conclusion is false. Equivalently, an inference is deductive just in case it is logically necessary that if the premises are true then the conclusion is true.

An argument encodes an inference or sequence of inferences. A deductive argument is one in which all of the inferences are (or purport to be) deductive. If all of the inferences are deductive, it follows that if the premises of the argument are true then the conclusion is also true. This relationship between the premises and conclusion are captured by the concept of a valid argument. Consider an argument having some number, say \( n \), premises: \( P_n, P_{n-1}, P_{n-2}, \ldots, P_2, P_1 \). Let the conclusion be, \( Q \). Then the argument looks as follows:

\[
\begin{align*}
P_1 \\
P_2 \\
P_3 \\
\vdots \\
P_n \\
\hline
\text{Therefore, } Q
\end{align*}
\]

Note that when we write the argument this way, we are ignoring the intermediate inferences. That is, we are focusing on the premises and
conclusion and abstracting from how the conclusion was gotten from the premises. We say that such an argument is valid if, and only if, it is logically impossible for its premises to be true while the conclusion is false. In other words, the argument is valid if, and only if, the inference it encodes is deductive.

Corresponding to the preceding argument we can construct the statement:

If $P_1$ and $P_2$ and $P_3$ and . . . and $P_n$, then $Q$.

This statement says that if the premises of the argument are all true then the conclusion is true. This statement is called the corresponding conditional of the argument. A conditional is any statement of the form “If $P$ then $Q$.” The corresponding conditional of an argument is the conditional in which $P$ says that the premises of the argument are all true and $Q$ is the conclusion of the argument. For example, the corresponding conditional of the following argument:

1. All Republicans are faithful to their spouses.
2. No one who is faithful to his or her spouse will ever get into political trouble.

Therefore, no Republican will ever get into political trouble.

is

If all Republicans are faithful to their spouses, and no one who is faithful to his or her spouse will ever get into political trouble, then no Republican will ever get into political trouble.

To say that an argument is valid is to say that it is logically necessary that if its premises are all true then its conclusion is true; that is, its corresponding conditional is logically necessary. Thus, for example, to say that the above argument is valid is just to say that the statement “If all Republicans are faithful to their spouses, and no one who is faithful to his or her spouse will ever get into political trouble, then no Republican will ever get into political trouble” is logically necessary. Hence we can talk about the validity of an argument by talking about the logical necessity of its corresponding conditional:

An argument is valid if, and only if, its corresponding conditional is logically necessary.

If an argument is valid and its premises are true, it follows that its conclusion is true. It should be emphasized, however, that if the premises of a valid argument are false, it does not follow that the conclusion is also false. The following is an example of a valid argument with (presumably) false premises and a true conclusion:
1. All Communists are bank managers.
2. All bank managers are followers of Karl Marx.

Therefore, all Communists are followers of Karl Marx.

Given a valid argument, the only time we can infer anything about whether its conclusion is true is when all its premises are true. If one or more of its premises are false, then its conclusion can be either true or false. The only combination of truth and falsity that is prohibited in a valid argument is all premises being true and the conclusion being false. Any other combination is possible.

5.3 Validity and Inferential Correctness

In section two it was asserted that logic studies the inferential correctness of arguments. That is not the same thing as validity, even if we focus exclusively on deductive arguments. This is because validity pertains only to the relationship between the premises and the conclusion of an argument, and ignores any intermediate steps whereby the conclusion is inferred from the premises. To illustrate, suppose we expand the above argument by adding an erroneous intermediate step and inferring the conclusion (incorrectly) from the intermediate step:

1. All Communists are bank managers.
2. All bank managers are followers of Karl Marx.

3. Therefore, all followers of Karl Marx are Communists. (from 1 and 2)

Therefore, all Communists are followers of Karl Marx. (from 3)

The premises and conclusion are unchanged from the previous argument, so the new argument is still valid. However, (3) does not follow from (1) and (2), and the conclusion does not follow from (3), so the argument is not inferentially correct. The argument is only valid "by accident", because two errors of reasoning cancelled out.

The term "sound" is sometimes used to refer to arguments that are not only valid, but also through and through inferentially correct. Unfortunately, that conflicts with the more customary use of "sound" to refer to arguments that also have true premises, so we will not adopt that terminology here. We might say instead that an argument is deductively correct if, and only if, every inference in it is deductive (and hence inferentially correct). If an argument is deductively correct, it follows that it is valid, but as we have seen, an argument can be valid without being deductively correct.

5.4 Explaining Deductive Inference Functionally

There is a school of thought, deriving from W. V. Quine, that is critical of the concepts of logical necessity and logical possibility. In effect, Quine's
objection was that we cannot make clear sense of these concepts by providing philosophical analyses of them. That is a rather strange objection, because it could be applied equally to most of our fundamental concepts. For example, philosophers have been no more successful at providing philosophical analyses of the concept of a physical object, or the concept of time. Does that mean that such concepts should be viewed as suspect? Surely not.

But regardless of whether the Quinean arguments are any good, they have convinced a number of philosophers. Without the concepts of logical necessity and logical possibility, we cannot explain deductive inference as above. This throws the foundations of logic into disarray. However, there is another way of characterizing deductive inferences. A deductive inference is one for which there are no (could be no) defeaters. This is to characterize deductive inference “functionally”, by describing how it works and what role it plays in human cognition. This approach takes defeasible inference to be the norm, and then characterizes deductive inference to be inference that is not subject to defeat.

This approach cries out for an explanation of what determines whether there are or could be any defeaters. That is to be explained by explaining where the structure of reasoning comes from. My own conviction is that it is dictated by the cognitive architecture of the human mind, which in turn is determined by our physiology. But this is a long story, and I will not pursue the details here.¹

This approach provides a way of resurrecting logical necessity from its Quinean coffin. The resurrection proceeds by turning the standard story on its head and explaining logical necessity in terms of deductive inference rather than the other way around. Deductive inference is characterized functionally, as sketched above, and necessary truths are just those that can be established purely deductively, without using any defeasible steps in our reasoning.

5.5 Defeasible Correctness

It was remarked above that most of our reasoning is defeasible rather than deductive. Typically, our reasoning will combine some defeasible inferences and some deductive inferences. Because defeasible inferences are not deductive, it is logically possible for the premises to be true but the conclusion false. This is why defeasible inferences can be defeated. For instance, when the inductive inference to “All swans are white” was defeated by finding black swans in Australia, the defeat resulted from discovering that the logical possibility of the premises being true but the conclusion false was more than a mere possibility—the conclusion actually was false despite the premises being true.

Arguments containing defeasible inferences cannot be expected to be

¹ Some of the details of this story are spelled out in John Pollock, Cognitive Carpentry, MIT Press, 1995; and in John Pollock and Joseph Cruz, Contemporary Theories of Knowledge, second edition, Rowman and Littlefield, 1999.
either valid or deductively correct. If they were they would be deductive arguments rather than defeasible arguments. But this does not mean that the arguments are somehow flawed. In the absence of defeaters, defeasible arguments can make it reasonable to believe their conclusions. They just do not logically guarantee that the conclusion is true given the premises. When all of the steps of a defeasible argument are inferentially correct, let us say that the argument itself is defeasibly correct. Defeasible correctness is to defeasible arguments as deductive correctness is to deductive arguments.

The use of defeasible arguments involves a trade-off. Defeasible arguments make their conclusions less certain than deductive arguments do. A deductive argument absolutely guarantees the truth of the conclusion if the premises are true. With a defeasible argument, it remains logically possible for the conclusion to be false even if the premises are true. The defeasible argument makes it reasonable to expect that the conclusion is true, but does not guarantee its truth. In this respect, we might think of defeasible arguments as inferior to deductive arguments.

But there is another sense in which they are superior, namely, they give us more information. A deductive argument can never lead to a conclusion that isn’t already implicit in the premises. In effect, what a deductive argument does is analyze the premises and reveal the consequences of what is being assumed. Defeasible arguments, on the other hand, are ampliative, in the sense that they can lead us to genuinely new information. For example, when we infer that no earthworms have wings, we can then deduce that earthworms in China don’t have wings, even though our original evidence for the conclusion that no earthworms have wings did not include any earthworms in China. Similarly, when I infer that the book on my desk is red because it looks red, the conclusion that the book is red expands upon my evidence and tells me something new about the world. It can be argued convincingly that the use of defeasible inference is essential to our being able to acquire knowledge of the world.\(^2\) Without defeasible reasoning we would never get beyond judgments about how things look to us.

6. Formal Logic

6.1 Argument Forms and Formal Validity

It was remarked above that an argument can be inferentially correct either because of the particular concepts that are employed in it or because of its general structure. When an argument is inferentially correct for the latter reason, we say that it’s form makes it correct.

The form of an argument is obtained by replacing some of its terms by schematic letters. For example, we might write a simple argument form as:

\(^2\) See Pollock and Cruz, *Contemporary Theories of Knowledge.*
1. All A are B.
2. No B is C.

Therefore, no A is C

Here A, B, and C are schematic letters whose places can be occupied by arbitrary predicates. What is important about this argument form is that every argument of this form will be inferentially correct, regardless of how we instantiate A, B, and C.

We defined an argument to be valid if, and only if, it is logically necessary that the conclusion is true provided the premises are true. We can extend the concept of validity to argument forms, saying that an argument form is valid if, and only if, every argument of that form is valid. Logic is often called “formal logic” because it concerns itself with the validity of argument forms rather than arguments. In other words, logic is concerned with the structural features of an argument that can make it valid (and hence make all other arguments with the same structure valid). The nonstructural aspects of arguments that can make them either valid or inferentially correct are also of philosophical interest, but they fall under the purview of epistemology rather than logic.

6.2 Statement Forms and Formal Necessity

Just as the form of an argument can make it valid, the form of a statement can make it logically necessary. For example

Either it is snowing or it is false that it is snowing

is logically necessary. However, the source of its necessity has nothing to do with meteorology. Any statement of the form

Either P or it is false that P

will be logically necessary. The source of the necessity is the form of the statement rather than the content of its constituents.

Statements do not have unique logical forms. Consider the statement

If John comes and Mary stays home then Bill be will unhappy.

This statement can be regarded as having at least two different forms:

If P and Q then R.
If S then R.

A statement might have one logical form all of whose instances are logically necessary, and another (more general) logical form some instances of which are not logically necessary. For example, consider the statement
If all featherless bipeds are men, and all men are mortal, then all featherless bipeds are mortal.

This statement has the logical form:

If all A are B and all B are C then all A are C.

Every statement of this form is logically necessary. But it also has the more general form:

If P then Q.

Statements of this form need not be logically necessary.

Let us say that a statement form is formally necessary if, and only if, every statement of that form is logically necessary. If a statement has a formally necessary form, then the statement is logically necessary. However, statements can also be logically necessary because of relationships between their constituents that are not captured by logical form. For instance, the statement

All bachelors are unmarried

is logically necessary, but it does not exhibit a form that is formally necessary.

6.3 Formal Necessity and Formal Validity

Logic studies both formal necessity and formal validity. There is an important connection between these two concepts. The validity of arguments (as opposed to argument forms) was defined in terms of the logical necessity of the corresponding conditional. We can also talk about the corresponding conditional of an argument form. This will be a statement form. For example, the corresponding conditional of the argument form

1. All A are B.
2. No B is C.

Therefore, no A is C

is

If all A are B and no B is C then no A is C.

In relating argument forms to their corresponding conditionals, the crucial observation is that a statement has the form of the corresponding conditional of the argument form if, and only if, it is the corresponding conditional of an argument of that form. For instance, the following argument and its corresponding conditional instantiate the preceding argument form and its corresponding conditional:
1. All A are B.  
2. No B is C.  

Therefore, no A is C 

If all A are B and no B is C then no A is C.

So a statement has the form of the corresponding conditional of the argument form if, and only if, it is the corresponding conditional of an argument of that form. An argument form is valid if, and only if, every argument of that form is valid. Any particular argument of that form is valid if, and only if, its corresponding conditional is logically necessary. So all arguments of a form are valid if, and only if, every statement instantiating the corresponding conditional is logically necessary. But the latter means, by definition, that the statement form that is the corresponding conditional of the argument form is formally necessary. So in sum:

An argument form is valid iff its corresponding conditional is formally necessary.

In light of this connection between argument forms and their corresponding conditionals, one way to study valid argument forms is to study the formal necessity of their corresponding conditionals. Most logical theories make use of this connection.

### 7. Logical Theories

#### 7.1 The Implicational Calculus

Logic is the study of valid argument forms and formally necessary statement forms. It was remarked above that there is no such thing as the form of a statement or argument. Statements and arguments can exemplify many different forms. Different logical theories are generated by focusing on different classes of statement and argument forms. To take a slightly simplistic example, consider all statement and argument forms that can be constructed using just “and”, “if ... then”, punctuation, and schematic letters standing for statements. Examples of statement forms constructed using these materials are:

- If P and Q then R.
- If, if P then Q and if Q then R, then if P then R.

As an illustration of how logic investigates logical forms, let us see what a logical theory of these logical forms would look like. Let us call the theory the implicational calculus. This is not a theory that people actually study, but
we can use it to illustrate how a logical theory proceeds.

Syntax

The first step in constructing a logical theory is generally to introduce an artificial representation of the statement forms studied. We could express them in English (augmented with schematic variables), but there are two problems with that. First, they would be hard to read. Second, English sentences are often ambiguous, and it is desirable to have a notation that avoids that ambiguity. For example, consider the expression we wrote above:

\[
\text{If, if P then Q and if Q then R, then if P then R.}
\]

This could mean several different things depending upon how the parts of the expression are grouped together. We could disambiguate it using parentheses for punctuation, in either of the following ways:

\[
\text{If } [(\text{if P then Q}) \text{ and } (\text{if Q then R})] \text{ then } (\text{if P then R}).
\]

\[
\text{If } [\text{if P then } [Q \text{ and } (\text{if Q then R})]] \text{ then } (\text{if P then R}).
\]

The first reading is probably the more natural, but the rules of English do not preclude the second reading. We want to avoid such ambiguities, and the best way to do it is to use parentheses for punctuation, just as we did above.

The English words “and” and “if ... then” also turn out to be ambiguous. For example, “P and Q” can express simple conjunction, saying merely that P and Q are both true, or it can express a temporal relation as in “He lay down and fell asleep”. As a simple conjunction, “P and Q” is equivalent to “Q and P”, but “He lay down and fell asleep” means something different from “He fell asleep and lay down”. To avoid this source of ambiguity, it is customary to introduce artificial symbols to replace the English words, and rule that these symbols have some particular interpretation of the English words as their meanings. The standard procedure is to symbolize “P and Q” as \((P \& Q)\), taking “\&” to express what I called “simple conjunction”. “If P then Q” is symbolized as \((P \rightarrow Q)\). Using these devices, we can symbolize the two readings of “If, if P then Q and if Q then R, then if P then R” as:

\[
[(P \rightarrow Q) \& (Q \rightarrow R)] \rightarrow (P \rightarrow R)
\]

\[
[P \rightarrow [Q \& (Q \rightarrow R)]] \rightarrow (P \rightarrow R).
\]

The expressions we can write in this way using “\&”, “\rightarrow”, schematic letters, and parentheses are called formulas of the implicational calculus.

In general, in constructing a logical theory, the first step is to define an artificial symbolism and define formulas of the theory to be the expressions of
the symbolism that express statement forms studied by the theory. The formulas will be constructed out of logical symbols, in this case "&" and "→", schematic letters, and parentheses. This part of the logical theory is its syntax.

**Semantics**

The next step in constructing a logical theory is to define what the formulas mean, i.e., to explain precisely what statement forms are expressed by any given formula. This explanation constitutes the semantics of the theory. The general procedure is to define the notion of an interpretation, and then explain the meaning of a formula by saying which interpretations make it true.

The definition of an interpretation varies a great deal from one logical theory to another. In general, interpretations are defined so that they abstract from the entire meaning and just specify the minimal amount of information needed to determine whether a formula is true or false. In the implicational calculus, it turns out that all we have to know to determine the truth value of a formula (i.e., whether it is true or false) is the truth values of the schematic letters out of which it is built. So we can define an interpretation of the implicational calculus to be an assignment of truth values (truth or falsity) to schematic letters. Other logical theories may have to include more or different information in their interpretations.

Given a definition of the interpretations of the logical theory, formulas receive their meanings by specifying the rules that determine under which interpretations they are true. The truth rule for "&" is obvious:

(P & Q) is true if, and only if, P is true and Q is true.

The truth rule for "→" is not obvious. The following rule is normally used:

(P → Q) is true if, and only if, it is not the case that Q is false but P is true.

This rule will be discussed at length in the next chapter. For now it will just be used as an illustration of the kind of rule one might adopt.

Repeated application of these rules enables us to determine the truth value of any formula of the implicational calculus under any interpretation. For example, suppose P is assigned truth, and Q and R are assigned falsity. Then consider the formula

\[(P \rightarrow Q) \& (Q \rightarrow R) \rightarrow (P \rightarrow R).\]

The truth rule for "→" makes (P → Q) and (P → R) false, but makes (Q → R) true. The rule for "&" then makes \[(P \rightarrow Q) \& (Q \rightarrow R)\] false. Finally, the rule for "→" makes the whole formula true.

Different logical theories use different techniques for formulating the semantics. In general, the desideratum is to describe the semantics in a
mathematically precise manner so that we can prove mathematical theorems about it. This need not be very difficult, as the example of the implicational calculus illustrates.

Formulas express statement forms. Once meanings have been assigned to formulas, we can ask which formulas are formally necessary. If we have done a good job giving a mathematical characterization of the meanings, it will be possible to prove a theorem that tells us which formulas are formally necessary. In fact, if we have defined interpretations appropriately, it will almost always be the case that we have the following theorem:

A formula is formally necessary if, and only if, it is true under every interpretation.

This theorem holds for the implicational calculus, although we will not prove it here.

Formulas true under every interpretation are said to be valid. This is unfortunate terminology, because it has no direct connection to the use of “valid” in talking about arguments. However, it is the standard terminology, so we will conform to convention and use it here. The reader is warned, however, not to confuse the two meanings of “valid”.

Defining validity for formulas in this way, the preceding theorem can be re-expressed as:

A formula is formally necessary if, and only if, it is valid.

Arguments

Our central interest in logic is usually with arguments rather than statements. The third step in constructing a logical theory is to specify what arguments can be constructed out of the formulas of the theory. Different theories can deal with arguments having different structures. The simplest arguments will be finite lists of formulas, constructed according to specified rules of inference. Rules of inference are rules telling us that certain formulas can be inferred from others. For example, in the implicational calculus we might use the following rule of inference in constructing arguments:

From P and (P → Q), infer Q.

This simple rule of inference has a long history, and accordingly has a Latin name, modus ponens (meaning “method of affirmation”).

Another simple rule that we might adopt is sometimes called adjunction:

From Q and R, infer (Q & R).

Let us also adopt a pair of rules that are collectively called simplification:

From (Q & R), infer Q.
From (Q & R), infer R.
Any bit of reasoning has to start with some premises, so we need a rule telling us that we can introduce formulas as premises. We normally want to allow the use of arbitrary additional premises in arguments so that we can study argument forms proceeding from those premises. Then we define a derivation in the implicational calculus to be a finite list of formulas each formula of which is either a premise or can be inferred from previous formulas in the list using the rules of inference we have chosen. Thus, for example, the following would be a derivation in the implicational calculus:

1. \((P \& Q)\) (premise)
2. \((P \rightarrow R)\) (premise)
3. \((Q \rightarrow S)\) (premise)
4. \(P\) (simplification, from 1)
5. \(Q\) (simplification from 1)
6. \(R\) (modus ponens, from 2 and 4)
7. \(S\) (modus ponens, from 3 and 5)
8. \((R \& S)\) (adjunction, from 6 and 7)

Derivations are argument forms. They are the argument forms studied by the logical theory in which they occur. A derivation is said to be a derivation of whatever formula appears on its last line from whatever premises are used in it. Thus the preceding derivation is a derivation of \((R \& S)\) from \((P \& Q)\), \((P \rightarrow R)\), and \((Q \rightarrow S)\).

We have seen that we can evaluate argument forms in terms of their corresponding conditionals. An argument form is valid if, and only if, its corresponding conditional is formally necessary. If the formal necessity of formulas corresponds to validity, then an argument form is valid if, and only if, its corresponding conditional is a valid formula. (Note the two distinct senses of “valid” in the preceding sentence.) Hence the semantics we have adopted for a logical theory provides the basis for investigating the validity of argument forms constructed within the theory.

To facilitate the investigation of argument forms within a logical theory, we define:

A set of formulas \(P_1, \ldots, P_n\) entails a formula \(Q\) if, and only if, the corresponding conditional \((P_1 \& \ldots \& P_n) \rightarrow Q\) is valid.

An argument is evaluated by asking whether its premises entail its conclusion. We typically want our derivations to satisfy two conditions:

\textit{Soundness:} If a formula is derivable from a set of premises, the premises should entail the formula.

\textit{Completeness:} If a set of premises entails a formula, the formula should be derivable from the premises.

The rules given above for the implicational calculus are sound but not complete. Notice that the use of “sound” in talking about systems of der-
7.2 The Propositional and Predicate Calculi

The implicational calculus is too simple to be of much real interest. Our first serious excursion into logic will consist of an investigation of the propositional calculus. The propositional calculus is much like the implicational calculus except that it adds logical symbols for “it is false that”, “or”, and “if and only if”. We can express some fairly complex reasoning within the propositional calculus, and the propositional calculus is still sufficiently simple to constitute a good introduction to logic. However, ultimately, it is not very useful. Most of the deductive reasoning we actually perform requires a richer symbolism.

After discussing the propositional calculus, we will turn to the predicate calculus. The predicate calculus is constructed by adding additional logical symbols to the propositional calculus. Specifically, we add ways of expressing “all” and “some”. It turns out that almost all of our deductive reasoning can be expressed in the predicate calculus. It is a very powerful theory. However, its semantics and systems of derivations are correspondingly more complex than for the propositional calculus. Fortunately, in studying the predicate calculus, we can build smoothly on what we learn from the propositional calculus. Both theories follow the general line illustrated by the implicational calculus, but the notion of an interpretation is more complex for the predicate calculus and the rules of inference are also more complex.

8. Summary

We defined the following terms:

Reasoning is inferentially correct if, and only if, given the premises, the reasoning supports the conclusion. This is independent of whether the premises are actually true.

A sound argument is one that is inferentially correct and also has true premises.

An inference is deductive if, and only if, it is logically impossible for the conclusion to be false when the premises are true.

Defeasible reasoning is reasoning that is inferentially correct but not deductive.

An argument is valid if, and only if, it is logically impossible for its conclusion to be false when its premises are true, i.e., if, and only if, the inference it encodes is deductive.
The corresponding conditional of an argument is the statement that if the premises are true then the conclusion is true.

It was then shown that we can evaluate the validity of an argument by looking at its corresponding conditional:

An argument is valid if, and only if, its corresponding conditional is logically necessary.

Turning to argument forms and statement forms, we defined:

An argument form is valid if, and only if, every argument of that form is valid.

A statement form is formally necessary if, and only if, every statement of that form is logically necessary.

It was then shown that we can evaluate the validity of an argument form by looking at its corresponding conditional:

An argument form is valid iff its corresponding conditional is formally necessary.

A logical theory focuses on a particular class of statement forms and investigates the formal necessity of statement forms in that class and the validity of argument forms constructed out of the statement forms.

A logical theory usually has three parts:

The syntax introduces an artificial symbolism for expressing the statement forms unambiguously, and defines what a formula is.

The semantics explains the meaning of the formulas. It does this by defining what an interpretation is and giving truth rules that determine which formulas are true relative to an interpretation. In the implicational calculus, an interpretation is an assignment of truth values to the schematic letters.

We defined:

A formula is valid if, and only if, it is true under every interpretation.

A set of formulas $P_1, \ldots, P_n$ entails a formula $Q$ if, and only if, the corresponding conditional $(P_1 \& \ldots \& P_n) \rightarrow Q$ is valid.

If we have defined interpretations appropriately, it will almost always be the case that we have the following theorem:
A formula is formally necessary if, and only if, it is true under every interpretation.

The system of derivations consists of a precise set of rules for constructing arguments using formulas. The arguments constructed in accordance with the rules are derivations. We typically want our derivations to satisfy two conditions:

**Soundness**: If a formula is derivable from a set of premises, the premises should entail the formula.

**Completeness**: If a set of premises entails a formula, the formula should be derivable from the premises.