Part One:

The Propositional Calculus
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The Syntax of the
Propositional Calculus

1. Introduction to the Propositional Calculus

The propositional calculus is an extension of the implicational calculus. The implicational calculus studied those statement and argument forms that can be constructed using just “and”, “if ... then”, punctuation, and schematic variables standing for statements. The formulas of the implicational calculus are not sufficiently expressive to be of much interest. However, if we augment them slightly we get a logical theory of much more interest. For this purpose we need to add “or”, “if and only if”, and “it is false that” to the list of constituents of statement forms. The resulting theory, the propositional calculus, is both of historical interest and of some practical interest. It is of historical interest because it is the first modern logical theory to have been studied carefully, and it is of practical interest because within the propositional calculus we can represent a number inference patterns that we employ regularly in our everyday reasoning.

2. The Sentential Connectives

The expressions “and”, “or”, “if ... then”, “if and only if”, and “it is false that”, are called sentential connectives, because they are used to connect sentences to form larger sentences. For example, the statement

If it is false that it is either raining in the mountains or snowing in the mountains then it is false that it is raining in Tucson

has the form

If it is false that (R or S) then it is false that T

where now R stands for “It is raining in the mountains”, S stands for “It is snowing in the mountains”, and T stands for “It is raining in Tucson”.

2.1 Logical Symbols

It was observed in chapter one that English punctuation is often ambiguous, and we introduced the use of parentheses to avoid the ambiguity. We follow that course again in the propositional calculus. It was also observed that the actual English words that we have called “the sentential connectives” are ambiguous, having several different usages. To avoid this ambiguity, we will follow the same strategy used in the implicational calculus. That is, we will introduce artificial symbols to express just that meaning of each sentential connective that we want to study in the propositional calculus.
The symbols we use to replace the sentential connectives are called the logical symbols of the propositional calculus. They are as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>~P</td>
<td>it is false that P</td>
</tr>
<tr>
<td>(P &amp; Q)</td>
<td>P and Q (or, both P and Q)</td>
</tr>
<tr>
<td>(P ∨ Q)</td>
<td>either P or Q</td>
</tr>
<tr>
<td>(P → Q)</td>
<td>if P then Q</td>
</tr>
<tr>
<td>(P ↔ Q)</td>
<td>P if and only if Q</td>
</tr>
</tbody>
</table>

We will have to elaborate on the explanations of meaning given in this table in order to avoid the ambiguities that are built into English, but let us postpone that for the moment.

Statements formed using the logical symbols have names, as follows:

- ~P is the negation of P.
- (P & Q) is the conjunction of P and Q, and P and Q are the conjuncts.
- (P ∨ Q) is the disjunction of P and Q, and P and Q are the disjuncts.
- (P → Q) is a conditional, P is the antecedent, and Q is the consequent.
- (P ↔ Q) is a biconditional. There are no standard names for the left and right side of a biconditional.

2.2 Sentential Letters

Using these symbols we can represent the forms of quite complex statements. We have been using capital letters as the schematic variables symbolizing the simple statements that form the parts of compound statements. We might encounter a statement having more than 26 sentential parts, so to accommodate this theoretical possibility we can also use capital letters with subscripts, such as P₁ or Q₇. In this book we will not actually encounter any sentences that long, but it may still be useful to have schematic variables with subscripts. When symbolizing the form of a statement, it is often helpful to use letters that remind us of what they are being used to symbolize. For example, we might symbolize “It is raining” as R, and “It is snowing” as S. However, the use of such mnemonics will occasionally leave us wanting to use the same letter to symbolize two different sentences. We can get around that by using subscripts.

It should be emphasized that the same letter with different subscripts, such as P and P₁, can be used to represent totally unrelated statements. The fact that P occurs in P₁ is just a coincidence. Capital letters, with or without subscripts, used this way as schematic variables are called sentential letters.
3. Formulas of the Propositional Calculus

Using sentential letters to stand for statements, and then combining them with logical symbols, we can express quite complex statement forms. The expressions we construct in this way are called \textit{formulas of the propositional calculus}. We can give precise rules for constructing formulas of the propositional calculus. The simplest formulas are simply sentential letters, with or without subscripts, such as \( P, Q, P_{13}, R_{129} \) and so forth. We call these \textit{atomic formulas} because they are the atoms from which more complicated formulas are constructed. Then all other formulas of the propositional calculus can be constructed by successive applications of the following rules:

1. An atomic formula is a formula.
2. If \( A \) is any formula, then \( \neg A \) is a formula.
3. If \( A \) and \( B \) are any formulas, then \( (A \& B) \) is a formula.
4. If \( A \) and \( B \) are any formulas, then \( (A \lor B) \) is a formula.
5. If \( A \) and \( B \) are any formulas, then \( (A \rightarrow B) \) is a formula.
6. If \( A \) and \( B \) are any formulas, then \( (A \leftrightarrow B) \) is a formula.

In writing the rules in this way, \( A \) and \( B \) are used as variables to stand for arbitrary formulas of the propositional calculus. Such variables are called \textit{metalinguistic variables}.

These rules define the concept of a formula of the propositional calculus. That is, an expression is a formula of the propositional calculus if, and only if, it can be constructed by repeated application of these rules. To illustrate, consider the formula

\[ (P \rightarrow (\neg Q \& R)) \]

We can construct this formula using the above six rules, as follows. We begin with the smallest parts and work outwards. By Rule 1, \( P, Q, \) and \( R \) are formulas. Then by Rule 2, \( \neg Q \) is a formula. By Rule 3, as \( \neg Q \) and \( R \) are both formulas, \( (\neg Q \& R) \) is a formula. Then by Rule 5, as \( P \) and \( (\neg Q \& R) \) are both formulas, \( (P \rightarrow (\neg Q \& R)) \) is a formula.

We can construct very complicated formulas using these rules. For example, consider the formula

\[ ((P \rightarrow Q_3) \leftrightarrow \neg (R_{17} \lor (R_{17} \& \neg P))) \]

Let us see how we would construct this formula using the six rules. Again, we begin with the smallest parts and work outwards. By Rule 1:

\( Q_3, R_{17}, R_{17} \)

are formulas because they are atomic formulas. As \( P \) and \( R_{17} \) are formulas, we can use Rule 2 to construct the formulas:

\( \neg P, \neg R_{17} \).
As \( \sim P \) is then a formula, by Rule 2 again,
\[
\sim \sim P
\]
is a formula. Then as \( R_{17} \) and \( \sim \sim P \) are both formulas, by Rule 3,
\[
(R_{17} \& \sim \sim P)
\]
is a formula. Then as \( R_4 \) and \( R_{17} \& \sim \sim P \) are both formulas, by Rule 4,
\[
(R_4 \lor (R_{17} \& \sim \sim P))
\]
is a formula. Then as \( Q_3 \) and \( R_4 \lor (R_{17} \& \sim \sim P) \) are both formulas, by Rule 6,
\[
(Q_3 \leftrightarrow (R_4 \lor (R_{17} \& \sim \sim P)))
\]
is a formula. Then by Rule 2,
\[
\sim (Q_3 \leftrightarrow (R_4 \lor (R_{17} \& \sim \sim P)))
\]
is a formula. We have already seen that \( \sim P \) is a formula, so by Rule 3,
\[
(\sim P \& \sim (Q_3 \leftrightarrow (R_4 \lor (R_{17} \& \sim \sim P))))
\]
is a formula. Then by Rule 2,
\[
\sim (\sim P \& \sim (Q_3 \leftrightarrow (R_4 \lor (R_{17} \& \sim \sim P))))
\]
is a formula, and then by Rule 2 again,\[
\sim (\sim P \& \sim (Q_3 \leftrightarrow (R_4 \lor (R_{17} \& \sim \sim P))))
\]
is a formula. This gives us the right side of the biconditional. Now working on the other side, as \( Q_3 \) and \( \sim R_4 \) are both formulas, by Rule 6,
\[
(Q_3 \leftrightarrow \sim R_4)
\]
is a formula. \( P \) is a formula, so by Rule 5,
\[
(P \rightarrow (Q_3 \leftrightarrow \sim R_4))
\]
is a formula. Both sides of the biconditional have now been constructed, and then by Rule 6,
\[
((P \rightarrow (Q_3 \leftrightarrow \sim R_4)) \leftrightarrow \sim (\sim P \& \sim (Q_3 \leftrightarrow (R_4 \lor (R_{17} \& \sim \sim P)))))
\]
is a formula. We can diagram the construction of this formula as follows:
Notice that whenever we construct a conjunction, disjunction, conditional, or biconditional, we enclose it in parentheses. In order to make formulas easier to read, parentheses are often replaced by brackets and braces. Thus we might write the above formula as

\[
\{\{P \rightarrow (Q \leftrightarrow \sim R_4)\} \leftrightarrow \sim\sim\left(\sim P \& \sim (Q \leftrightarrow (R_4 \lor (R_{17} \& \sim\sim P)))\right)\}.
\]

By doing this we can tell at a glance which left parenthesis or bracket goes with which right parenthesis or bracket; thus, we can see more easily what the formula means. But it is to be emphasized that the expression we obtain by using brackets and braces is not literally a formula—it is an abbreviation for a formula.

Formulas with large numbers of nested parentheses can be hard to read. A useful trick for parsing such formulas is to connect matching parentheses with lines:

\[
\begin{align*}
((P \rightarrow (Q \leftrightarrow \sim R_4)) &\leftrightarrow \sim\sim\left(\sim P \& \sim (Q \leftrightarrow (R_4 \lor (R_{17} \& \sim\sim P)))\right)) \leftrightarrow \sim\sim\left(\sim P \& \sim (Q \leftrightarrow (R_4 \lor (R_{17} \& \sim\sim P)))\right) \\
\end{align*}
\]

Atomic formulas are the simplest formulas. Let us say that a formula is \textit{molecular} if it is not atomic. Then molecular formulas are those formulas that contain logical symbols.

Sometimes we will want to talk about one formula being a \textit{part} of another formula. This just means that the first occurs somewhere in the second. For example, \(P, Q, R, (P \rightarrow Q), \sim (P \rightarrow Q), \) and \(\sim\sim (P \rightarrow Q) \lor R\) are all parts of the formula \(\sim\sim (\sim\sim (P \rightarrow Q) \lor R)\). Let us also adopt the convention of saying that a formula is part of itself. Then

\[
\sim\sim (\sim\sim (P \rightarrow Q) \lor R)
\]

is also a part of \(\sim\sim (P \rightarrow Q) \lor R\). The \textit{atomic parts} of a formula are those
parts of the formula that are atomic. Thus P, Q, and R are the atomic parts of \( \neg [(P \to Q) \lor R] \), while \((P \to Q), \neg (P \to Q), [\neg (P \to Q) \lor R] \) and \( \neg [\neg (P \to Q) \lor R] \) are parts that are not atomic.

With the exception of negations and atomic formulas, all formulas must have outer parentheses. However, the outer parentheses are not necessary for making the formula unambiguous. The outer parentheses are only needed when the formula is used in constructing a larger formula. For example,

\[ P \to Q \]

is unambiguous. However, to form its negation we cannot write

\[ \neg P \to Q. \]

That would be the abbreviation for \((\neg P \to Q)\), whereas what we want is \((\neg P \to Q)\). So for the sake of enhanced readability, we could allow ourselves to abbreviate formulas by omitting outer parentheses. But if we do that we must take care to supply the omitted parentheses when combining formulas with other formulas. And it must be borne in mind that this is strictly an informal abbreviation, and what we write in that way is technically not a formula.

**Exercises**

A. Show how the following formula of the propositional calculus can be constructed using Rules 1 through 6. Diagram the construction as was done on page 30.

\[ ((P \to Q) \leftrightarrow (\neg(Q \to P) \lor \neg(Q \land \neg P))) \]

B. Construct a table for each of the following expressions indicating: (1) whether it is a formula of the propositional calculus; (2) if it is a formula, whether it is atomic or molecular; (3) if it is a molecular formula, whether it is a negation, conjunction, disjunction, conditional or biconditional; (4) if it is a formula, what its atomic parts are; (5) if it is a formula, what its parts are:

1. P
2. \( \neg \neg P \)
3. \( P \lor Q \)
4. \((P \lor Q) \to \neg Q) \)
5. \((\neg(P \leftrightarrow Q)) \)
6. \((P \lor Q) \to R) \)
7. \(\neg((Q \& R) \leftrightarrow (\neg Q \lor \neg R)) \)
8. \((P \lor P) \)
9. \((P \& \neg P) \)
10. \((P \to Q) \leftrightarrow (\neg Q \leftrightarrow P) \)
4. Symbolizing Statement Forms

Formulas of the propositional calculus can be used to symbolize quite complex statement forms. For example, consider the statement

If he comes we will have the party at his house, and if he doesn't come then we will have the party at Jones' house.

Letting $P$ be the statement “He comes”, $Q$ the statement “We will have the party at his house”, and $R$ the statement “We will have the party at Jones’ house”, we can first symbolize the statement as

If $P$ then $Q$, and if it is false that $P$ then $R$.

This in turn can be symbolized as

If $P$ then $Q$, and if $\neg P$ then $R$

and then as

$(P \rightarrow Q) \land (\neg P \rightarrow R)$

and finally as

$[(P \rightarrow Q) \land (\neg P \rightarrow R)]$.

As another example, consider

If Jones needs money, then either he will reduce prices or he will apply to the bank for a loan.

Letting $P$ be “Jones needs money”, $Q$ be “He will reduce prices”, and $R$ be “He will apply to the bank for a loan”, this statement can be symbolized, first as

If $P$ then either $Q$ or $R$,

then as

If $P$ then $(Q \lor R)$,

and finally as

$[P \rightarrow (Q \lor R)]$.

There are two things that should be noticed about the preceding examples. The first is that in symbolizing relatively complex statements like “If he comes we will have the party at his house, and if he doesn't come then we will have the party at Jones' house”, we begin by symbolizing the smallest parts, and then we construct the successively larger parts one at a time until we finally get the whole statement. The symbolization went as follows:
If \( P \) then \( Q \), and if \( \text{it is false that } P \) then \( R \)

\[(P \rightarrow Q) \text{ and } \neg P \rightarrow R\]

\[(P \rightarrow Q) \text{ and } \neg P \rightarrow R\]

\[
[(P \rightarrow Q) \& (\neg P \rightarrow R)]
\]

It is always best to begin by symbolizing the smallest parts of a statement first, and then constructing the symbolization of the successively larger parts one at a time in terms of the symbolization of their constituents. It is unwise to try to symbolize an entire statement in one step if the statement is at all complicated.

The second thing to notice is the use of parentheses. Whenever we symbolize a part of the statement—unless that part is a negation—we enclose it in parentheses to keep it separate from the other parts of the statement. The parentheses take the place of commas and other grammatical devices of English. We must be careful to include the parentheses, or the resulting symbolization will be ambiguous. Consider the two statements

It is not true that Jones has a girlfriend and his wife is going to divorce him

and

It is not true that Jones has a girlfriend, and his wife is going to divorce him.

These obviously mean quite different things. The first denies that it is true both that Jones has a girlfriend and that his wife is going to divorce him, whereas the second says that Jones does not have a girlfriend, but his wife is going to divorce him anyway. These two statements would be symbolized as \( \neg(P \& Q) \) and \( (\neg P \& Q) \) respectively. But if we omitted the parentheses in symbolizing these statements and just wrote \( \neg P \& Q \), it would be indeterminate which of these two different statements were meant.

It is not necessary to enclose a negation (a statement of the form \( \neg P \)) in parentheses, because "\( \neg \)" does not really connect sentences—it acts on a single sentence. But for any of the other sentential connectives it is necessary to use parentheses each time the connective is used.

Exercises

Symbolize the forms of the following statements, paying careful attention to the use of commas to force (sometimes unnatural) interpretations. Use the sentential letters indicated:

1. If it rains today, the ground will be wet and we will not be able to
have a picnic.  (R G P)
2. It is not true that if it is cloudy then it will rain.  (C R)
3. If the sun shines in the morning it will rain, and if the sun does not shine in the morning then it will not rain.  (S R)
4. If we get plenty of sunshine, then if it rains the flowers will grow.  (S R F)
5. It is false that, there is a woman in the next room if and only if Jim said there is.  (W S)
6. It is false that there is a woman in the next room, if and only if, Jim said there is.  (W S)
7. Either the entrails will contain cockroaches or they will not, and if they do then the gods are angered, and if they do not then Venus will be in apposition to Jupiter.  (E G V)
8. If she is an acrobat or she is a clown, then she lives in that trailer.  (A C T)
9. Harry will go to the bank today if and only if the market drops, and if Harry goes to the bank today the supervisors will hide, and if the supervisors hide and Harry does not go to the bank today then Emmett will lose his shirt on the stock market.  (H M S E)
10. If Francis Bacon wrote Hamlet and Shakespeare wrote Macbeth, then either Shakespeare was Bacon or the theater manager was a crook.  (H M S C)

5. Conditionals

The sentential connective that has the greatest number of importantly different uses is “if ... then”. On any of its uses, it is said to express a conditional, but there are several different kinds of conditionals.

5.1 Indicative Conditionals and Subjunctive Conditionals

The most important distinction is between indicative conditionals and subjunctive conditionals. Indicative conditionals have the form “If it is true that P then it is true that Q”. By contrast, subjunctive conditionals have the form “If it were true that P then it would be true that Q”. These have importantly different meanings. Ernest Adams illustrates the difference with the pair of sentence:

If Oswald didn’t shoot Kennedy, someone else did.

If Oswald hadn’t shot Kennedy, someone else would have.

1 “The logic of conditionals”, Inquiry 8 (1965), 166-197.
The former is clearly true. After all, Kennedy got shot. But there is no reason to think that the latter is true. Had Oswald been thwarted, Kennedy would most likely have survived.

Most uses of subjunctive conditionals are to express what are called counterfactual conditionals. A counterfactual conditional is about what would have happened had something that is actually false been true. By contrast, indicative conditionals are normally used to talk about what is true if something we are not sure about is true.

There is an extensive literature on subjunctive conditionals, stemming largely from Robert Stalnaker and David Lewis. My own account of subjunctive conditionals can be found in my book The Foundations of Philosophical Semantics. There is no consensus on the analysis of subjunctive conditionals, but it is agreed by everyone that however they are to be analyzed, they are not properly expressed by the "\( \rightarrow \)" of the propositional calculus. Somewhat surprisingly, indicative conditionals have proved more recalcitrant to analysis than subjunctive conditionals, and there is little agreement about whether the "\( \rightarrow \)" of the propositional calculus properly expresses indicative conditionals.

5.2 Material Conditionals

In the propositional calculus, "\( \rightarrow \)" expresses what is called the material conditional. The material conditional (P \( \rightarrow \) Q) is defined to mean (\( \neg P \lor Q \)). In the next chapter, we will see that this is equivalent to \( \neg (P \land \neg Q) \), so the latter could equally be used as the definition of the material conditional. For example, if the sentence "If it rains then the crops will grow" is interpreted as expressing a material conditional, it means "Either it won’t rain or the crops will grow". Equivalently, it means "It is false that it will rain but the crops won’t grow".

There is an argument which seems to show that the material conditional is the same as the indicative conditional. Consider an indicative conditional "If P then Q". The argument purports to show that it is true if, and only if, (\( \neg P \lor Q \)) is true. For the sake of concreteness, let P be "It rains" and Q be "The crops will grow". The argument proceeds in two stages. First we suppose that "If P then Q" is true and show that it follows that (\( \neg P \lor Q \)) is true. Then we suppose that (\( \neg P \lor Q \)) is true and show that it follows that "If P then Q" is true:

1. Suppose that "If P then Q" is true. So if it rains (i.e., P is true) then Q is true. If it does not rain, then \( \neg P \) is true. But either it will not rain or it will rain. So either it will not rain, and then \( \neg P \) will be true, or else it will rain, and then Q will be true. Thus, no matter what happens, either \( \neg P \) will be

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true or $Q$ will be true; that is, $(\neg P \lor Q)$ will be true. So we see that if the statement "If $P$ then $Q$" is true, then $(\neg P \lor Q)$ is true.

2. In order to show that "If $P$ then $Q$" and $(\neg P \lor Q)$ are true under exactly the same circumstances, it must also be shown that if $(\neg P \lor Q)$ is true then "If $P$ then $Q$" is true. So let us suppose that $(\neg P \lor Q)$ is true. A disjunction $(A \lor B)$ is true if, and only if, at least one of its disjuncts is true; that is, if, and only if, either $A$ is true or $B$ is true. Thus if $(\neg P \lor Q)$ is true, then either $\neg P$ is true or $Q$ is true. To show that "If $P$ then $Q$" is true, we can reason as follows:

If $P$ is true, then $\neg P$ cannot be true. But by hypothesis, either $\neg P$ or $Q$ must be true. So if $P$ is true then $Q$ must be true; that is, "If $P$ then $Q$" is true.

Thus it has been shown that if $(\neg P \lor Q)$ is true, then "If $P$ then $Q$" is true.

So we have an argument that seems to show that "If $P$ then $Q$" and $(\neg P \lor Q)$ are true in exactly the same circumstances. In other words, the indicative conditional is properly analyzed as being the same as the material conditional. However, as persuasive as the argument seems to be, the conclusion is somewhat counterintuitive. The difficulty is that for a disjunction to be true, all that is required is that at least one disjunct is true. For instance, the disjunction "That is either gold or fool’s gold" is true if it is gold and true if it is fool’s gold. This has the consequence that $(\neg P \lor Q)$ is true if $\neg P$ is true, i.e., if $P$ is false. But that is equivalent to saying that the material conditional $(P \rightarrow Q)$ is true whenever its antecedent $P$ is false. To test this claim, consider an example. Suppose I have some influence with the state Lottery Commissioner, but not enough to affect the outcome of the lottery. I am also scrupulously honest and know that I would never use my influence for any purpose. So I know that I will not use my influence with the Lottery Commissioner. But from that I can infer that the material conditional "If I use my influence with the Lottery Commissioner, you will win the lottery" is true. If we understand this as an ordinary indicative conditional, there is some temptation to deny that it is true, because my influence is not so great that I could actually influence the outcome of the lottery.

Examples like this are often taken to show that the material conditional is not the same thing as the indicative conditional. However, we must not be too quick to draw this conclusion. First, note that the most natural interpretation of "If I use my influence with the Lottery Commissioner, you will win the lottery" is as a subjunctive conditional, not an indicative conditional. That is, what would ordinarily be taken as being asserted is that if I were to use my influence, you would win the lottery. The distinction between subjunctive and indicative conditionals is in terms of meaning, not in terms of the words used. The words are ambiguous and can be used to express either an indicative conditional or a subjunctive conditional.

Still, one can employ the sentence "If I use my influence with the Lottery Commissioner, you will win the lottery" to express an indicative conditional, and under the circumstances described there is something odd
about such a conditional. It does not seem reasonable to assert this just because I know that I will not use my influence.

However, its being unreasonable to assert the conditional is not the same thing as its being false. Under the circumstances described, it would be equally odd to assert the disjunction “Either I won’t use my influence or you will win the lottery”. The explanation for this may lie in what are called “implicatures”.

5.3 Conversational Implicatures

The concept of an implicature is due to Paul Grice. Logicians talk about one statement implying another. That just means that it is logically necessary that if the first statement is true then the second statement is true. So understood, implication is a relation between statements. We also talk about people implying things. One way to imply something is to make a statement that implies it. But sometimes we can imply something by making a statement that does not itself have that implication. Instead of the statement having the implication, it is the act of making the statement under those circumstances that has the implication. If my hostess asks me how I like the dessert and I reply, “It is very pretty”, I imply that I don’t like the taste. The statement that it is pretty does not imply that I don’t like the taste. My making the statement that it is pretty, under those circumstances, is what carries the implication. Similarly, if a professor critiques a student’s paper by saying, “It is very well typed”, he implies that he does not like its content. In cases like this, in which it is the act of making the statement rather than the statement itself that carries the implication, we say that it is a conversational implicature.

Grice observed that there are rules governing the correct use of language that do more than determine meanings. Some of these rules govern conversation. One such rule is something like, “Don’t say less than you know that is relevant”. For example, if you are shown a metal bar and asked what it is made of, if you know that it is made of titanium then there would be something wrong with your replying, “It is made of either titanium or aluminum”. By saying the latter you imply that you do not know which it is made of. This is a conversational implicature.

One of the interesting characteristics of conversational implicatures is that they can be “cancelled” by explicitly denying them. For instance, if I say, “It is made of either titanium or aluminum, and I know which but I am not going to tell you”, then I no longer imply that I don’t know which, nor do I contradict myself. On the other hand, logical implications cannot be cancelled. If I say, “There are more than four apples in that barrel, but I don’t mean to imply that there are more than three”, I am simply talking nonsense or contradicting myself.

Now let us apply this to material conditionals. It seems that what is wrong with my asserting, “Either I won’t use my influence or you will win the lottery”, is that I know that I won’t use my influence, and so I am saying less than I know that is relevant. This is analogous to saying, “It is made of

either titanium or aluminum”, when I know that it is made of titanium. I am implying that I don’t know that I won’t use my influence, and so implying that the disjunction is true because of some sort of connection between the two disjuncts that makes one true if the other is false. Similarly, if I say, “If I use my influence then you will win the lottery”, I am implying that I might use my influence, and that the conditional is true because of some connection between the antecedent and consequent such that if I did use my influence then the consequent would be true.

Perhaps what is wrong with asserting an indicative conditional just because the antecedent is false is not that the conditional isn’t true but rather than it is a pointless thing to say. The point of asserting a conditional is to license an inference. If we discover that the antecedent is true then we can infer that the consequent is also true. Conversely, if we discover that the consequent isn’t true, we can infer that the antecedent isn’t true. If we already know that the antecedent isn’t true, then the first inference can never occur, and the second inference is pointless because we already know that its conclusion is true. Hence it is pointless to assert the conditional.

Grice’s observations suggest that there is nothing wrong with identifying the indicative conditional with the material conditional. There are cases in which the material conditional is true but it would be unreasonable to assert the indicative conditional. However, Grice’s observations indicate that it would be equally unreasonable to assert that material conditional. Truth alone is not sufficient to make something assertable. The rules of conversation impose other requirements as well, and the problematic conditionals fail to satisfy those other requirements. That, rather than falsity, seems to be their failing.

5.4 Using Material Conditionals

As we have seen, there are arguments to the effect that the indicative conditional is the same as the material conditional. We should not regard the matter as settled, however. This is still an issue being debated in philosophical logic. Even if it were eventually decided, to everyone’s satisfaction, that these two conditionals are not the same, we would continue to use “→” to express the material conditional in the propositional calculus. This is for two reasons. First, it is at least often true that we can express what we want to express using material conditionals. Of course we cannot express every conditional claim using material conditionals. For example, we cannot express subjunctive conditionals in that way. But this just means that the propositional calculus is not expressively complete, not that what it can express is useless. Second, the material conditional turns out to be easy to study. As we will see in the next chapter, it has a simple truth rule that makes it possible to investigate its logical properties using the same semantics that is required for “and”, “or”, and “it is false that”. These different logical concepts form a nicely unified package that can be studied together, and the result is the propositional calculus.
6. Paraphrasing

Sometimes it is necessary to paraphrase a statement before it can be symbolized. Consider the statement

John and Joe both came to the party.

We want to symbolize this as a conjunction, but it cannot be symbolized directly because “and” stands between two names rather than between two sentences. Before we can symbolize it we must paraphrase it so that the sentential connective connects two sentences:

John came to the party and Joe came to the party.

Then letting P be “John came to the party” and Q be “Joe came to the party”, we can symbolize it as (P & Q).

Another example of such paraphrasing is found in the following statement:

If either Kennedy or Khrushchev had been weaker willed concerning either Berlin or Cuba, the cold war would have turned into a hot war.

This must be paraphrased first as

If either Kennedy had been weaker willed concerning either Berlin or Cuba, or Khrushchev had been weaker willed concerning either Berlin or Cuba, then the cold war would have turned into a hot war.

Then this must be paraphrased again as

If either Kennedy had been weaker willed concerning Berlin or Kennedy had been weaker willed concerning Cuba, or Khrushchev had been weaker willed concerning Berlin or Khrushchev had been weaker willed concerning Cuba, then the cold war would have turned into a hot war.

Finally then, this can be symbolized as [(P ∨ Q) v (R ∨ S)] → T.

Other kinds of paraphrasing may also be necessary. There are expressions in English such as “unless”, “but”, “if”, “only if”, “neither … nor”, that are much like the sentential connectives. Whenever these occur in a statement, the statement must be paraphrased to replace them by sentential connectives. For example,

Neither Joe came to the party nor John came to the party

means the same thing as

Joe didn’t come to the party and John didn’t come to the party

and so it must be paraphrased in that way. In general, “Neither P nor Q” can be paraphrased as (¬P & ¬Q). Equivalently, it can be symbolized as ¬(P v Q).

“But” is like “and” but emphasizes a contrast because the two conjuncts.
We may say “He came but he didn’t like it” rather than “He came and he
didn’t like it” merely to indicate that the two conjuncts do not go naturally
together. This emphasis makes no difference for logic, so we can symbolize
“P but Q” as (P & Q).

The behavior of the expressions “if” and “only if” is somewhat
surprising. There is a strong temptation to identify “P if Q” with “If P then
Q”, but in fact it should be the other way around. That is, “P if Q” means
the same thing as “If Q then P”. Consider the statement “The crops will be
destroyed if there is a flood”. To say this is not to say that the crops might
not be destroyed anyway, for example, by a drought. But the statement “If
the crops are destroyed there is a flood” precludes the possibility of them
being destroyed by a drought as opposed to a flood. Therefore it cannot be
a proper paraphrase of “The crops will be destroyed if there is a flood”. The
proper paraphrase is “If there is a flood then the crops will be destroyed”.
In general, “P if Q” can be symbolized as (Q → P).

“P only if Q” works just the other way around. It means “If P then
Q”. Consider the statement “The crops will be destroyed only if there is a
flood”. This means that the only way the crops can be destroyed is by a
flood, and hence if the crops are destroyed then there must have been a
flood. This then means “If the crops are destroyed then there is a flood”. Note
that “P if and only if Q” is just the conjunction of “P if Q” and “P only
if Q”, and thus that (P ↔ Q) is equivalent to [(P → Q) & (Q → P)].

One further expression that can be paraphrased in terms of the sentential
connectives is “unless”. “P unless Q” can be paraphrased as (~Q → P).
Suppose we want to paraphrase the statement “We will go to the beach
unless it rains”. This is the same thing as saying “If it doesn’t rain then we
will go to the beach” that is (~Q → P).

One thing to beware of in symbolizing the forms of statements is that
there are other uses of some of the sentential connectives in which they do
not have their ordinary meaning. We noted in chapter one that “and”
sometimes means “and then” rather than simply “and”. In its ordinary use,
“P and Q” means “It is true that P, and it is true that Q”. On this reading,
“P and Q” means the same thing as “Q and P”. For example, “This is red
and that is white” means the same thing as “That is white and this is red”.
But consider the use of “and” in “He lay down and fell asleep”. This
clearly does not mean the same thing as “He fell asleep and lay down”. The
former means “He lay down and then fell asleep”. This temporal use
of “and” is not among the logical concepts the propositional calculus deals
with. In particular, it cannot be symbolized simply as “&”.

**Exercises**

Symbolize the forms of the following statements, using the sentential letters
indicated:

1. Neither Jack nor Jim will come unless Mary comes. (A I M)
2. We will not get there on time unless we speed, but if we speed we
   will not get there at all. (T S G)
3. We can get the door open only if we use an acetylene torch on it, but then the door will be ruined. (O A R)

4. The river will not overflow its banks unless we either have an early thaw or heavy rains, but we will not have heavy rains. (O E H)

5. Unless we have a flat tire, we can get there on time if we speed, but we will have a flat tire if we speed. (F T S)

6. Neither Jack nor Jim will come if Mary comes, unless Joan and Mary both come. (A I M O)

7. Jeremy will get a Mercedes Benz for Christmas only if he does not offend Santa Claus, but Jeremy will offend Santa Claus if he does not believe in him, and Jeremy does not believe in Santa Claus. (M O B)

8. Rain is imminent. (you pick the sentential letters on this one)

9. John will not come unless Jim comes, and Jim will not come if Jeffrey comes, but Jeffrey will only come if John does not come. (O I E)

10. It will rain if the barometer drops, but if it rains it will cool off later, and it will not cool off later. (R B C)

7. Summary

The propositional calculus studies statement forms and argument forms that can be constructed out of “and”, “or”, “it is false that”, “if ... then”, and “if and only if”. These are the sentential connectives, and they are symbolized using the logical symbols &, ∨, ~, →, and ↔.

Formulas of the propositional calculus were defined to be those expressions that can be constructed using the six rules on page 29. For all but atomic formulas and negations, formulas must have outer parentheses.

In symbolizing statement forms, it is best to begin with the smallest parts and work outwards.

Some statements must be paraphrased before they can be symbolized:
- “Neither P nor Q” is symbolized as ~(P ∨ Q) or as (~P & ~Q).
- “P but Q” is symbolized as (P & Q).
- “P if Q” is symbolized as (Q → P).
- “P only if Q” is symbolized as (P → Q).
- “P unless Q” is symbolized as (~Q → P).

Subjunctive conditionals are about what would be the case if something else were the case.

Indicative conditionals are about what is the case if something else is the case.

The material conditional (P → Q) is defined to mean (~P ∨ Q). It is unclear whether this is a proper symbolization of the indicative conditional.
**Practice Problems**

Symbolize the forms of the following statements, using the sentential letters indicated:

1. If Meacham is impeached (I) or convicted (C), then unless he resigns before he is removed from office (R), he will be unable to run in the recall election (U).

2. If Robertson wins Florida (F) and Jackson wins Tennessee (T), then if Hart loses in Georgia (G), Gephart will win in Indiana (I) if he is still in the race (R).

3. If Victoria and the Kaiser were cousins (C), then Germany and England were friendly (F), in which case World War II never happened (W).

4. If you take the test (T) and fail (F) you will be in trouble with the university (U), but if you take the test and pass you will be in trouble with the other students (S), and if you don’t take the test you will be in trouble with the Dean (D).

5. If Henry is in pursuit of Oriental mysteries (P), then if he finds the dragon lady (L) and the Foo Dog (D), he will be unable to emulate Charlie Chan (C) unless either the Dragon Lady is benevolent (B) or the Foo Dog is broken (K).

6. If the mining venture proves successful (S) but Jacob backs out too early (E), then Jacob’s bank account will suffer (U) and his greedy parents will disown him (D) unless he turns to a life of crime (C).

7. If the job is finished on time (F), we will be able to eat lunch on the lawn (L), but if it isn’t finished on time we will have to eat lunch in the cellar (C), and in that case we will have to share our lunch with the mice (M).

8. Neither England nor France will be competitive in downhill skiing (E,F) unless Switzerland beats Austria (B), but Switzerland won’t beat Austria unless Schwartz breaks his leg (S).

9. If the plant will die (D) unless it gets water (W), but it won’t get water if the sun shines (S) (unless there is a miracle (M)), then the plant will die.

10. If Marilyn Monroe married Guy Lombardo (M) and they had a son named “Ronnie” (R), he too could be president (P) if he moved to Old Tucson (T) and became a cowboy movie star (C).