Phil 202
Sample Final Exam

You may use your book and notes in answering these questions.

I. Symbolize the following statements, using the symbols indicated:

1. If anyone takes this exam, the professor will have to grade it, and if the professor has to grade this exam he will be angry, in which case anyone who takes the exam will get a low grade on it. (P: “is a person”; T: “(1) takes (2)”; G: “(1) will have to grade (2)”; A: “will be angry”; L: “(1) will get a low grade on (2)”; e: this exam; p: the professor) (25 points)

2. Anyone who is taller than someone is taller than someone who isn’t taller than anyone. (P: “is a person”; T: “(1) is taller than (2)” (20 points)

II. Show that the following formulas are consistent by constructing interpretations under which they are true:

3. $(\forall x)[(\exists y)Hxy \& (\exists z)\neg Hxz]$ (15 points)

4. $(\exists x)(\forall y)(Hxy \& Hyx)$ (15 points)

III. Prove the following metatheorem:

5. For any formulas P and Q of the predicate calculus, $P \vdash Q$ iff $\neg Q \vdash \neg P$.

IV. For each of the following, give an annotated derivation of the formula below the line from the formulas (if any) above the line.

6. no premises (20 points)

$$\neg (\exists x)(\forall y)(Hxy \& \neg Hxx)$$

7. $(\forall x)(\forall y)[Fx \to (Gy \to Hxy)]$ (25 points)

$$\neg (\forall y)(\exists x)(Fx \& \neg Hxy) \to \neg Gy$$

8. $(\exists x)(\forall y)(\exists z)(\forall w)Gxyzw$ (25 points)

$$\neg (\forall y)(\exists x)(\forall w)(\exists z)Gxyzw$$
Phil 202
Solutions to Sample Final Exam

You may use your book and notes in answering these questions.

I. Symbolize the following statements, using the symbols indicated:

1. If anyone takes this exam, the professor will have to grade it, and if the professor has to grade this exam he will be angry, in which case anyone who takes the exam will get a low grade on it. (P: “is a person”; T: “(1) takes (2)”; G: “(1) will have to grade (2)”;
A: “will be angry”; L: “(1) will get a low grade on (2)”; e: this exam; p: the professor) (25 points)

$$((\exists x)(P(x) \land T(x)) \rightarrow G(e, p)) \land (G(e, p) \rightarrow A(p)) \land [(\forall y)(P(y) \land T(y)) \rightarrow L(y)]$$

2. Anyone who is taller than someone is taller than someone who isn’t taller than anyone. (P: “is a person”; T: “(1) is taller than (2)” (20 points)

$$\forall x(\exists y)(P(x) \land T(x, y)) \rightarrow (\exists y)(P(y) \land T(y, x) \land \neg \exists z(P(z) \land T(z, x)))$$

II. Show that the following formulas are consistent by constructing interpretations under which they are true:

3. $$\forall x(\exists y)H(x, y) \land (\exists z)\neg H(x, z)$$ (15 points)

Domain = {1,2}, H: {{1,1}, {2,2}} (e.g., H means “[1] is identical to [2]”)

4. $$\exists x(\forall y)(H(x, y) \land H(y, x))$$ (15 points)

Domain = {1}, H: {{1,1}}

III. Prove the following metatheorem:

5. For any formulas P and Q of the predicate calculus, P $$\vdash$$ Q iff $$\neg Q$$ $$\vdash$$ $$\neg P$$.

Suppose P $$\vdash$$ Q. Then (P $$\rightarrow$$ Q) is valid, i.e., true under every interpretation.. By contraposition, (P $$\rightarrow$$ Q) is equivalent to ($$\neg Q$$ $$\rightarrow$$ $$\neg P$$), so the latter is also true under every interpretation. But that means that $$\neg Q$$ $$\vdash$$ $$\neg P$$.

IV. For each of the following, give an annotated derivation of the formula below the line from the formulas (if any) above the line.

6. no premises (20 points)

$$\neg(\exists x)(\forall y)(H(x, y) \land \neg H(x, x))$$

1. $$\neg(\exists x)(\forall y)(H(x, y) \land \neg H(x, x))$$

2. $$\neg(\forall y)(H(x, y) \land \neg H(x, x))$$ for interest 1 by (E21).

3. $$\neg(\forall y)(H(x, y) \land \neg H(a, a))$$ for interest 2 by UG

(1) 1. (H(x, y) $$\land$$ H(x, x)) premise (for reductio)

(1) 2. (H(y, y) $$\land$$ H(x, x)) II6, 1

(1) 3. H(x, x) II, 2
4. \(\neg Haa\) I2, 2

5. \(\neg (\forall y)(H_{xy} \& \neg Haa)\) R, 3, 4 discharges interest 3

6. \((\exists x)(\forall y)(H_{xy} \& \neg Hxx)\) UG, 5 discharges interest 2

7. \(\neg (\exists x)(\forall y)(H_{xy} \& \neg Hxx)\) E21, 6 discharges interest 1

7. \[(\forall x)(\forall y)[F_x \to (G_y \to H_{xy})]\] (25 points)

\[
(\forall y)[(\exists x)(F_x \& \neg H_{xa}) \to \neg G_y]
\]

1. \(\forall x)(\forall y)[F_x \to (G_y \to H_{xy})]\) premise

(1) 1. \((\forall y)[(\exists x)(F_x \& \neg H_{xa}) \to \neg G_y]\)

(1) 2. \((\exists x)(F_x \& \neg H_{xa}) \to \neg G_y\) for interest 1 by UG

2. \(\exists x)(F_x \& \neg H_{xa})\) premise

1, 2 3. \(\neg G_y\) for interest 2 by C, using premise (2).

3. \(F_b \to \neg H_{ba}\) I1, 3

4. \(\neg H_{ba}\) I2, 3

5. \(\exists x)(F_x \& \neg H_{xa})\) premise (for EI)

6. \((\forall y)[F_b \to (G_a \to H_{ba})]\) I16, 1

7. \([F_b \to (G_a \to F_b)]\) I16, 6

1, 3 8. \(G_a \to H_{ba}\) I9, 4, 7

1, 3 9. \(\neg G_a\) I10, 5, 8

1, 2 10. \(\neg G_a\) EI, 9 discharges interest 3

1, 3 11. \([\exists x](F_x \& \neg H_{xa}) \to \neg G_y]\) C, 2, 10 discharges interest 2

1, 1 12. \((\forall y)[(\exists x)(F_x \& \neg H_{xa}) \to \neg G_y]\) UG, 11

8. \((\exists x)(\forall y)(\exists z)(\forall w)G_{xyzw}\) (25 points)

\[
(\forall y)(\exists x)(\forall w)(\exists z)G_{xyzw}
\]

1. \((\exists x)(\forall y)(\exists z)(\forall w)G_{xyzw}\) premise

1. \((\forall y)(\exists x)(\forall w)(\exists z)G_{xyzw}\)

2. \((\forall y)(\exists z)(\forall w)G_{xyzw}\) premise (for EI)

2. \((\forall y)(\exists x)(\forall w)(\exists z)G_{xyzw}\) for interest 1 by EI

3. \((\exists x)(\forall w)(\exists z)G_{xyzw}\) for interest 2 by UG

4. \((\forall w)G_{abcw}\) premise (for EI)

4. \((\exists x)(\forall w)(\exists z)G_{xyzw}\) for interest 3 by EI

5. \((\forall w)(\exists z)G_{abcw}\) for interest 4 by I17

6. \((\exists z)G_{bcd}\) for interest 5 by UG

7. \(G_{abcd}\) for interest 6 by I17

5. \(G_{bcd}\) I16, 4 discharges interest 7

6. \((\exists z)G_{bcd}\) I17, 5 discharges interest 6

7. \((\forall w)(\exists z)G_{abcw}\) UG, 6 discharges interest 5

8. \((\exists x)(\forall w)(\exists z)G_{xyzw}\) I17, 7 discharges interest 4

9. \((\exists x)(\forall w)(\exists z)G_{xyzw}\) EI, 8 discharges interest 3

10. \((\forall y)(\exists x)(\forall w)(\exists z)G_{xyzw}\) UG, 9 discharges interest 2

11. \((\forall y)(\exists x)(\forall w)(\exists z)G_{xyzw}\) EI, 10 discharges interest 1