Phil 202
Sample Third Exam

You may use your book and notes in answering these questions.

I. Symbolize the following statements, using the symbols indicated:

1. If anyone is infected, they will become very sick unless someone injects them with the serum. (P: “is a person”; I: “is infected”; S: “will become very sick”; J: “(1) injects (2) with (3)”; s: the syrum) (15 points)

\[(\forall x)((Px \land Ix) \rightarrow [-\exists y(Py \land Jyxs) \rightarrow Sx])\]

2. If anyone is infected, someone caught the disease from someone who was in Afghanistan. (P: “is a person”; I: “is infected”; C: “(1) caught (2) from (3)”; A: “was in Afghanistan”; d: the disease) (15 points)

\[\{(\exists x)(Px \land Ix) \rightarrow (\exists y)[Py \land (\exists z)((Pz \land Az) \land Cydz)]\}\]

II. Show that the following formulas are consistent by constructing interpretations under which they are true:

3. \[\[(\forall y)Hyy \land (\exists x)\neg(\forall y)Hxy\] (15 points)

   domain = \{1,2\}
   H: \{<1,1>,<2,2>\}

4. \[(\forall x)[Fx \rightarrow (\exists y)(\exists z)(\forall w)(Hxyz \leftrightarrow \neg Hzyw)]\] (15 points)

   domain = \{1\} (or anything else)
   F: \emptyset
   H: any set of ordered pairs of elements of the domain.

III. Prove the following metatheorem:

5. For any formula P of the predicate calculus, \(\vdash P \iff \neg P \vdash P\).

Suppose \(\vdash P\). Then P is true under every interpretation. So in particular, P is true under every interpretations under which \(\neg P\) is true, i.e., \(\neg P \vdash P\).
Conversely, suppose \(\neg P \vdash P\). If there is an interpretation under which \(\neg P\) is true, then it makes P true. But that is impossible, so there is no interpretations under which \(\neg P\) is true. I.e., P is true under every interpretation, and so \(\vdash P\).